Agenda

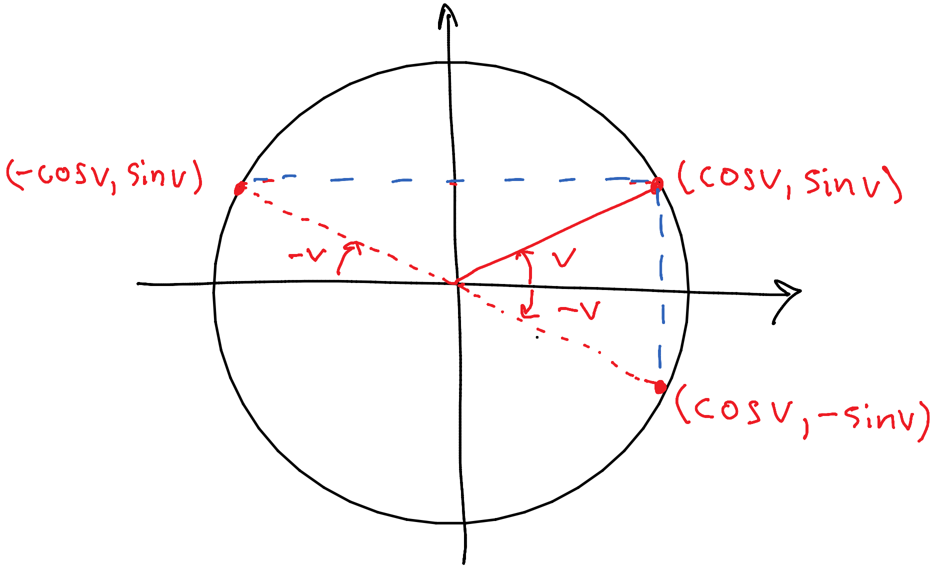
den 19 november 2020 12:38

- Ekvationerna

$\cos(u) = \cos(v)$, $\sin(u) = \sin(v)$

har lösningarna

$$\begin{cases} u = v + n2\pi \\ \text{eller} \\ u = -v + n\cdot 2\pi \end{cases} \qquad \begin{cases} u = v + n\cdot 2\pi \\ \text{eller} \\ u = \pi - v + n\cdot 2\pi \end{cases} \qquad n \in \mathbb{Z}$$



$$\left[\begin{array}{l} \cos(\pm v + n\cdot 2\pi) = \cos(v) \\ \sin(v + n\cdot 2\pi) = \sin(v) \end{array}\right. \quad \& \quad \sin(\pi - v + n\cdot 2\pi) = \sin(v)$$

- $a\cos(v) + b\sin(v) = c\sin(v + \varphi)$

$c = \sqrt{a^2 + b^2}$ (Amplituden)

$$\varphi = \begin{cases} \arctan\left(\frac{a}{b}\right) & \text{om } b > 0 \\ \arctan\frac{a}{b} + \pi & \text{om } b < 0 \end{cases}$$

(Fasvinkeln)

eller $\begin{cases} \cos\varphi = \frac{b}{c} \\ \sin\varphi = \frac{a}{c} \end{cases}$

Uppgift 1

den 12 november 2020   22:14

Bestäm alla lösningar x i intervallet [0, 2 pi]

Sådana att sin(3x) = cos (2x)

Lösning

Vi har $\sin 3x = \cos\left(\frac{\pi}{2} - 3x\right)$

Då får vi att

$$\cos\underbrace{\left(\frac{\pi}{2} - 3x\right)}_{u} = \cos \underbrace{2x}_{v}$$

Dvs 1) $\frac{\pi}{2} - 3x = 2x + n \cdot 2\pi, \quad n \in \mathbb{Z}$

eller 2) $\frac{\pi}{2} - 3x = -2x + n \cdot 2\pi, \quad n \in \mathbb{Z}$

Vi har nu två enkla ekvations-system att lösa.

$$\begin{cases} \sin\left(\frac{\pi}{2} - x\right) = \cos x \\ \cos\left(\frac{\pi}{2} - x\right) = \sin x \end{cases}$$

1) $\frac{\pi}{2} - 3x = 2x + n \cdot 2\pi$ (lös ut x)

$n \in \mathbb{Z}, \; n = 0, \pm 1, \pm 2, \ldots$

$\frac{\pi}{2} = 5x + n \cdot 2\pi$

$5x = \frac{\pi}{2} - n \cdot 2\pi$

$$\underline{\underline{x = \frac{\pi}{10} - n \cdot \frac{2\pi}{5}}}$$

2) $\frac{\pi}{2} - 3x = -2x + n \cdot 2\pi$

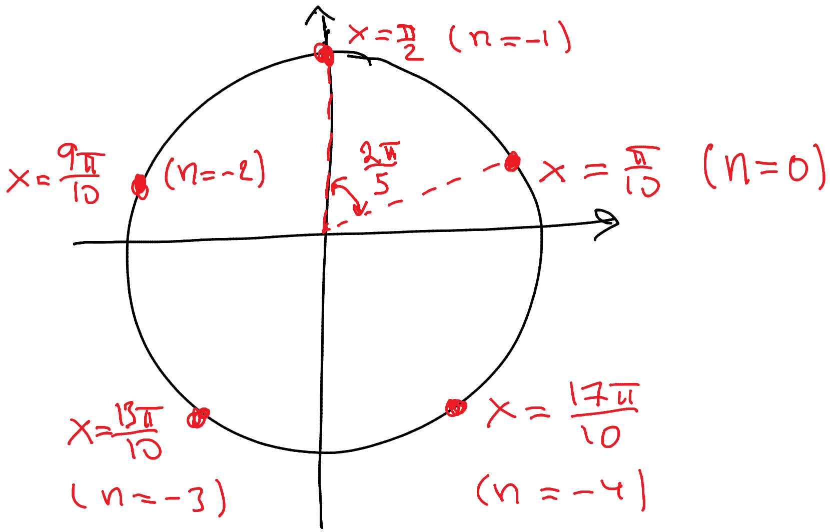
$\frac{\pi}{2} = x + n \cdot 2\pi$

$x = \underline{\underline{\frac{\pi}{2} - n \cdot 2\pi}}$,

Kommentar:

Sammanställningen visar att ekvation 2) inte ger några nya lösningar.

Vi söker enbart lösningar i intervallet $0 \leq x \leq 2\pi$. Enklast är att sammanställa dessa i enhetscirkeln:



$\frac{\pi}{10} = 18^\circ$, $\frac{2\pi}{5} = 72^\circ$

Svar:

$x = \frac{\pi}{10}, \frac{\pi}{2}, \frac{9\pi}{10}, \frac{13\pi}{10}, \frac{17\pi}{10}$

Breakout 1

den 19 november 2020 12:25

Bestäm alla lösningar till ekvationen

$a)\ \cos(2x) = \frac{\sqrt{3}}{2}, \qquad -\pi \le x \le \pi$

$b)\ \tan(3x) = 1, \qquad x \in R$

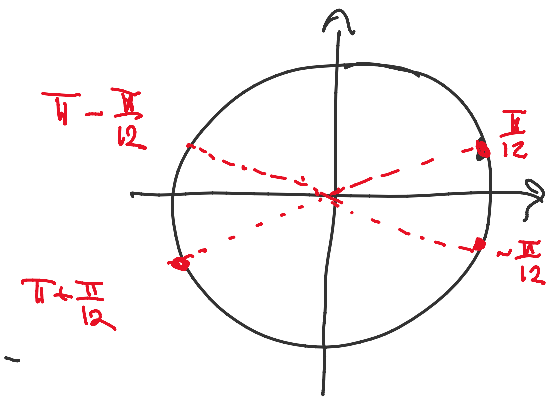
a) $\frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$

Vi löser ekv

$\cos 2x = \cos\frac{\pi}{6}$

$2x = \pm\frac{\pi}{6} + n\cdot 2\pi, \ n \in \mathbb{Z}$

$x = \pm\frac{\pi}{12} + n\cdot\pi$

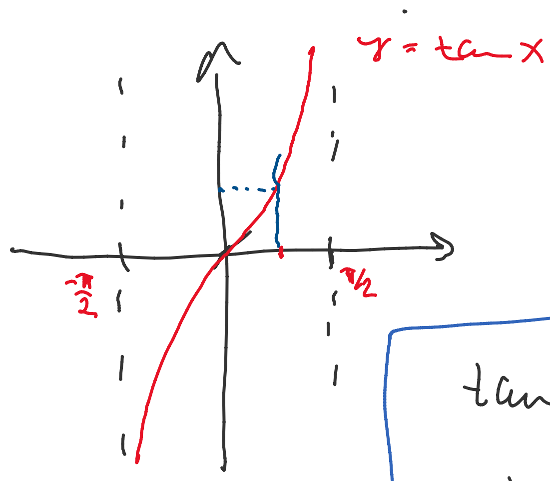


Svar: $-\frac{\pi}{12}, \frac{\pi}{12}, \frac{-11\pi}{12}, \frac{11\pi}{12}$

b) $\tan(3x) = 1 = \tan\frac{\pi}{4}$

$$3x = \frac{\pi}{4} + n\pi, \quad n \in \mathbb{Z}$$

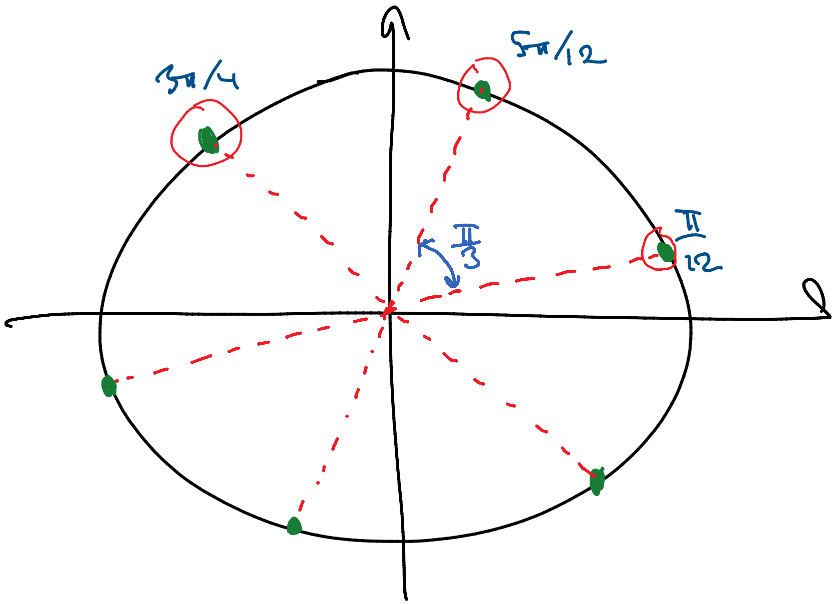
$$x = \frac{\pi}{12} + n \cdot \frac{\pi}{3}, \quad n \in \mathbb{Z}$$



$$\boxed{\tan(x + n\cdot\pi) = \tan x}$$

$\tan(x)$ har perioden π

Svar: $x = \frac{\pi}{12} + n \cdot \frac{\pi}{3}, \quad n \in \mathbb{Z}$



Vi kan också skriva lösningarna som

$x = \frac{\pi}{12} + n \cdot \pi$ eller $x = \frac{5\pi}{12} + n \cdot \pi$ eller $x = \frac{3\pi}{4} + n \cdot \pi$

där n i vardera fallet är ett godtyckligt heltal

Uppgift 2

den 19 november 2020 13:05

7. En lösning till ekvationen (6p)

$$2\tan x + \frac{1}{\cos x} = 1$$

är $x = 0$, men det finns ju fler. Bestäm *alla* lösningar.

L: $\left(x \neq \frac{\pi}{2} + n\pi\right)$ $\left(\tan x = \frac{\sin x}{\cos x}\right)$

Multiplicera med $\cos x$:

$$\underbrace{2\tan x \cdot \cos x}_{= \sin x} + 1 = \cos x$$

$$2\sin x + 1 = \cos x$$

$$\underbrace{\cos x - 2\sin x}_{= \sqrt{5}\cdot \sin(x+\varphi)} = 1 \qquad (*)$$

$$\sin(x+\varphi) = \frac{1}{\sqrt{5}}$$

$$x + \varphi = \arcsin\frac{1}{\sqrt{5}} + n\cdot 2\pi$$

eller $\qquad\qquad (n \in \mathbb{Z})$

$$x + \varphi = \pi - \arcsin\frac{1}{\sqrt{5}} + n\cdot 2\pi$$

(*)

$VL = \cos x - 2\sin x = c \cdot \sin(x+\varphi)$

där $c = \sqrt{1^2 + (-2)^2} = \sqrt{5}$

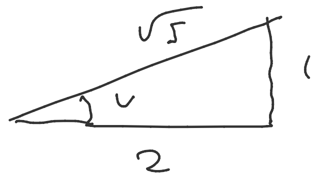
$\varphi = \arctan\left(\frac{1}{-2}\right) + \pi$

$= \underbrace{-\arctan\frac{1}{2}}_{\approx 26.5^\circ} + \pi \approx 153.5^\circ$

[här är $a = 1$, $b = -2 < 0$]

(**)

$\underbrace{\arcsin\frac{1}{\sqrt{5}}}_{= \arctan\frac{1}{2}} \approx 26.5^\circ$ (se figur)



$\begin{cases} \tan v = \frac{1}{2}, & v = \arctan\frac{1}{2} \\ \sin v = \frac{1}{\sqrt{5}} & v = \arcsin\frac{1}{\sqrt{5}} \end{cases}$

Dvs

$$x = -\varphi + \arcsin\frac{1}{\sqrt{5}} + n\cdot 2\pi = \arctan\frac{1}{2} - \pi + \arcsin\frac{1}{\sqrt{5}} + n\cdot 2\pi \overset{**}{=} -\pi + 2\arctan\frac{1}{2} + n\cdot 2\pi$$

eller

$$x = \pi - \varphi - \arcsin\frac{1}{\sqrt{5}} + n\cdot 2\pi = \arctan\frac{1}{2} - \arcsin\frac{1}{\sqrt{5}} + n\cdot 2\pi \overset{**}{=} n\cdot 2\pi$$

$n \in \mathbb{Z}$

Breakout 2

den 19 november 2020 12:49

$$\sin u = \cos\left(\frac{\pi}{2} - u\right) = \cos\left(u - \frac{\pi}{2}\right)$$

Skriv som en sinusfunktion

$\sqrt{3}\cos(x) - \sin(x)$

Lösning

$$-\sqrt{3}\cos x - \sin x$$

$$c = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{4} = 2$$

$$\varphi = \arctan\frac{\sqrt{3}}{-1} + \pi$$

$$= -\arctan\sqrt{3} + \pi$$

$$= -\frac{\pi}{3} + \pi = \frac{2\pi}{3}$$

$$a\cos v + b\sin v = c\cdot\sin(x+\varphi)$$

$$a = \sqrt{3}, \quad b = -1$$

$$\varphi = \arctan\frac{a}{b} \quad \text{om } b > 0$$

$$\varphi = \arctan\frac{a}{b} + \pi \quad \text{om } b < 0$$

$$\arctan(-x) = -\arctan x$$

$$\begin{cases} \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \\ \cos\frac{\pi}{3} = \frac{1}{2} \end{cases} \qquad \tan\frac{\pi}{3} = \sqrt{3}$$

$$\arctan\sqrt{3} = \frac{\pi}{3}$$

$$\tan(\underbrace{\arctan\sqrt{3}}_{u}) = \sqrt{3}$$

$$\tan u = \frac{\sin u}{\cos u}$$

Svar $\sqrt{3}\cos x - \sin x = \underline{\underline{2\cdot\sin\left(x + \frac{2\pi}{3}\right)}}$

$c = 2$ är Amplitud, $\varphi = \frac{2\pi}{3}$ är fasvinkel

Dugga 3

den 19 november 2020 15:43

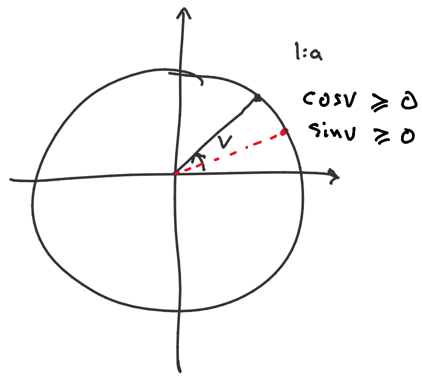
Man vet att $\tan v = \frac{21}{20}$ och att v ligger i första kvadranten.

Beräkna exakt:

a) $\sin \frac{v}{2} =$

b) $\cos \frac{v}{2} =$

c) $\cot \frac{v}{2} =$



$$\sin \frac{v}{2} = \underset{(-)}{+} \frac{\sqrt{1-\cos v}}{2}$$

[$v \in$ 1:a kvadranten

$\Rightarrow \frac{v}{2} \in$ 1:a kvadranten

$\Rightarrow \sin \frac{v}{2} > 0$]

$$\begin{cases} x = \sin v \\ y = \cos v \end{cases}$$

1) $\frac{x}{y} = \frac{\sin v}{\cos v} = \tan v = \frac{21}{20}$

$x > 0,\ y > 0$

2) $x^2 + y^2 = 1$

$x = \frac{21}{20} y$ sätt in i 2)

$$\left(\frac{21}{20}\right)^2 y^2 + y^2 = 1$$

$$21^2 y^2 + 20^2 y^2 = 20^2$$

$$(\underbrace{21^2 + 20^2}_{441 + 400}) y^2 = 20^2$$

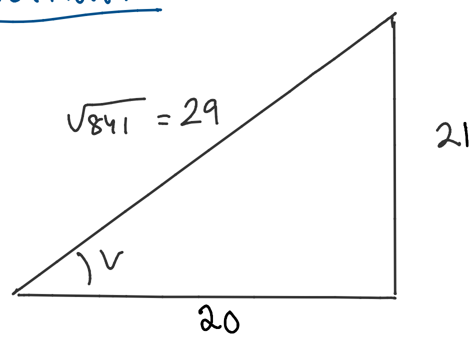
$$y^2 = \frac{400}{841}, \quad y = \frac{20}{\sqrt{841}} = \underline{\underline{\frac{20}{29}}}$$

Svar får vi via formeln

$$\sin \frac{v}{2} = \frac{1}{2}\sqrt{1 - \frac{20}{29}} = \frac{1}{2}\sqrt{\frac{9}{29}}$$

$$\cos \frac{v}{2} = \frac{1}{2}\sqrt{1 + \frac{20}{29}} = \frac{1}{2}$$

Alternativ



Figuren med kateterna 20 och 21 och hypotenusa $\sqrt{20^2 + 21^2} = \sqrt{841} = 29$

Får vi direkt

$$\underline{\underline{\cos v = \frac{20}{29}}} \quad \left(\text{och } \sin v = \frac{21}{29}\right)$$