8.10 Givet $y^4 + 4xy^2 - 4x^2 = 28$ (*)

beräkna $y''(1)$ om $y(1) < 0$ (1)

$x = 1$ Vad blir y?

Stoppa in $x = 1$ i (*)

$y^4 + 4y^2 - 4 = 28$

$y^4 + 4y^2 - 32 = 0$

$t = y^2 \Rightarrow t^2 + 4t - 32 = 0$

$t = -2 \pm \sqrt{4 + 32} = -2 \pm 6 = \begin{cases} 4 \\ -8 \end{cases}$

$\sqrt{36} = 6$

$y = \pm\sqrt{t} = \pm\sqrt{4} \overset{(1)}{=} -2$

$\Rightarrow (x, y) = (1, -2)$

$y^4 + 4xy^2 - 4x^2 - 28 = 0$

derivera 1 gång

$4y^3 \cdot y' + 4y^2 + 8yy'x - 8x = 0$

$\underbrace{y'}_{f}\underbrace{(4y^3 + 8yx)}_{g} = 8x - 4y^2 \Rightarrow y' = \left.\frac{8x - 4y^2}{4y^3 + 8yx}\right|_{(x,y)=(1,-2)} =$

$f' = y'' \quad g' = 12y^2y' + 8(y'x + y) \quad = \frac{8 - 16}{-32 - 16} = \frac{-8}{-48} = \frac{1}{6}$

$y''(4y^3 + 8yx) + y'(12y\,y' + 8(y'x + y)) = 8 - 8yy'$

$y'' = \frac{8 - 16yy' - 12y(y')^2 - 8(y')^2x}{4y^3 + 8yx} = \frac{2 - y'(4y - 3yy' - 2yx)}{y^3 + 2yx} =$

$= \{x = 1, y = -2, y' = \frac{1}{6}\} = -\frac{53}{216}$

Nästa uppgift vid 14:15

4. Bestäm D_f, asymptoter och extrempunkter till

$$f(x) = \frac{x^2}{\sqrt{x^2+4x+3}}$$

$x^2+4x+3 \le 0 \Rightarrow$ problem

$$x = -2 \pm \sqrt{4-3} = -2 \pm 1 = \begin{cases} -1 \\ -3 \end{cases}$$

$$D_f = \mathbb{R} \setminus [-3,-1] = (-\infty,-3) \cup (-1,\infty)$$

$$\lim_{x \to -3_-} f(x) = \text{"}\frac{9}{0}\text{"} = \infty$$

$$\lim_{x \to -1_+} f(x) = \text{"}\frac{1}{0}\text{"} = \infty$$

$\Rightarrow$ lodräta asymptoter $\begin{cases} x = -3 \\ x = -1 \end{cases}$

$y = kx + m$

$$k_\pm = \lim_{x \to \pm\infty} \frac{f(x)}{x}$$

$$m_\pm = \lim_{x \to \pm\infty} f(x) - k_\pm \cdot x$$

$$k = \lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{x}{|x| \underbrace{\sqrt{1+\frac{4}{x}+\frac{3}{x^2}}}_{\to 1}} = \begin{cases} 1 & (x \to \infty) \\ -1 & (x \to -\infty) \end{cases}$$

$$m_+ = \lim_{x \to \infty} f(x) - x = \lim_{x \to \infty} \frac{x^2 - x\sqrt{x^2+4x+3}}{\sqrt{x^2+4x+3}}$$

$$\left[\frac{\cancel{x^4} - x^2(\cancel{x^2}+4x+3)}{\sqrt{x^2+4x+3}\,(x^2 + x\sqrt{x^2+4x+3})} = \frac{-4x^3 - 3x^2}{x^2 |x| \sqrt{1+\dots}\,(1+\sqrt{1+\dots})} = \frac{-4-\frac{3}{x}}{\sqrt{1+\dots}\,(1+\sqrt{1+\dots})} \right]$$

$$\lim_{x \to \infty} \frac{-4 - \overbrace{\frac{3}{x}}^{\to 0}}{\underbrace{\sqrt{1+\frac{4}{x}+\frac{3}{x^2}}}_{\to 1}\,\left(1+\underbrace{\sqrt{1+\frac{4}{x}+\frac{3}{x^2}}}_{\to 1}\right)} = \frac{-4}{2} = -2$$

$y = x - 2$ $\quad x = -3$

$y = -x + 2$ $\quad x = -1$

asymptoter

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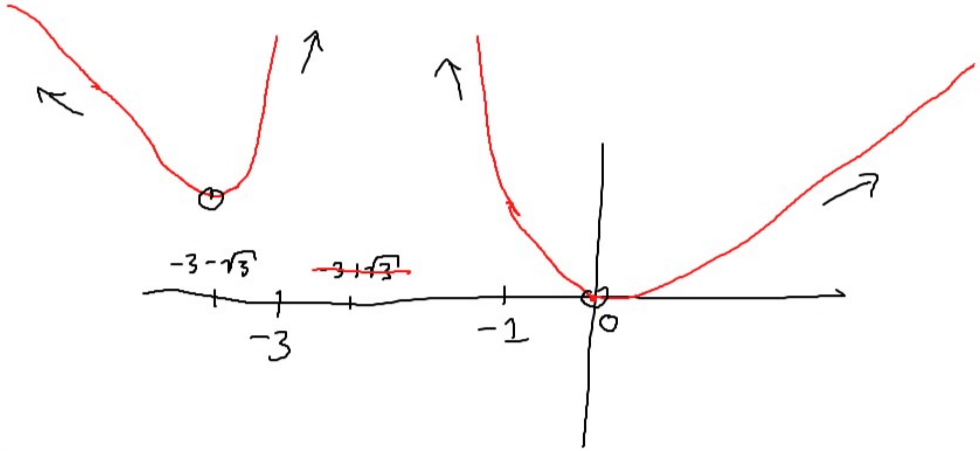
4. Bestäm D_f, asymptoter och extrempunkter till

$$f(x) = \frac{x^2}{\sqrt{x^2+4x+3}}$$

$D_f = \mathbb{R} \setminus [-3,-1] = (-\infty,-3) \cup (-1,\infty)$



$$f'(x) = \frac{2x\sqrt{x^2+4x+3} - x^2 \frac{1}{2}\frac{2x+4}{\sqrt{x^2+4x+3}}}{x^2+4x+3} = 0$$

$$2x\sqrt{x^2+4x+3} - x^2\frac{x+2}{\sqrt{x^2+4x+3}} = 0$$

$$2x(x^2+4x+3) - x^2(x+2) = 0$$

$$2x^3+8x^2+6x - x^3 - 2x^2 = x^3+6x^2+6x = 0$$

$$x(x^2+6x+6) = 0$$

$$x = 0, \quad x = -3 \pm \sqrt{9-6} = -3 \pm \sqrt{3}$$

lok min i

$x = 0$

$x = -3-\sqrt{3}$

asymptoter

$x=-3 \quad x=-1$

$y = x-2 \qquad y = -x+2$