Blandade uppgifter

Exempel 1: primitiv till $\frac{x^2}{1+x^2}$

$$\frac{x^2}{1+x^2} = \frac{x^2+1-1}{x^2+1} = \frac{x^2+1}{x^2+1} - \frac{1}{x^2+1} = 1 - \frac{1}{x^2+1}$$

primitiv till 1 är x

primitiv till $\frac{1}{x^2+1}$ är $\arctan x$

$$F(x) = x - \arctan x + C$$

$$F(0) = 3$$

$$F(0) = 0 - \arctan 0 + C = C = 3$$

4.9 , 95

$v_0 = 50$ km/h

$v_1 = 80$ km/h

$t = 5s$

$a = ?$

$$v_1 = v_0 + at$$

$$a = \frac{v_1 - v_0}{t} = \frac{30 \text{ km/h}}{5s}$$

$$30 \text{ km/h} = \frac{30000 \text{ m}}{3600 \text{ s}}$$

4.5 , 24

$$f(x)=\sqrt{x^2+x}-x \;,\qquad x^2+x\geq 0,\quad x(x+1)$$

		-1		0	
x	$-$		$-$		$+$
$x+1$	$-$		$+$		$+$
	$+$		$-$		$+$

$$|x|=\begin{cases} x, & x\geq 0\\ -x, & x<0\end{cases}$$

i) $D_f=(-\infty,-1]\cup[0,\infty)$

ii) $$\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)=\lim_{x\to\infty}\frac{\left(\sqrt{x^2+x}-x\right)\left(\sqrt{x^2+x}+x\right)}{\sqrt{x^2+x}+x}=\lim_{x\to\infty}\frac{x^2+x-x^2}{\sqrt{x^2+x}+x}=$$

$$=\lim_{x\to\infty}\frac{x}{|x|\sqrt{1+\frac{1}{x}}+x}=\lim_{x\to\infty}\frac{x}{x\sqrt{1+\frac{1}{x}}+x}=\lim_{x\to\infty}\frac{1}{\sqrt{1+\underbrace{\tfrac{1}{x}}_{\to 0}}+1}=\frac{1}{2}$$

$$\lim_{x\to-\infty}\frac{x}{|x|\sqrt{1+\frac{1}{x}}+x}=\lim_{x\to-\infty}\frac{x}{-x\sqrt{1+\frac{1}{x}}+x}=\lim_{x\to-\infty}\frac{1}{-\sqrt{1+\underbrace{\tfrac{1}{x}}_{\to 0}}+1}=\infty$$

4.5, 24 (forts)

$$\lim_{x\to -1-} \left(\sqrt{x^2+x} - x\right) = 1 \qquad \sqrt{x^2} = |x|$$

$$\lim_{x\to 0+} \left(\sqrt{x^2+x} - x\right) = 0$$

$$k = \lim_{x\to -\infty} \frac{\sqrt{x^2+x} - x}{x} = \lim_{x\to -\infty} \frac{\overset{\overset{-x}{\shortparallel}}{|x|}\sqrt{1+1/x} - x}{x} = \lim_{x\to -\infty} \frac{-\sqrt{1+1/x} - 1}{1} = -2$$

$$m = \lim_{x\to -\infty} \left(\sqrt{x^2+x} - x + 2x\right) = \lim_{x\to -\infty} \left(\sqrt{x^2+x} + x\right) = \lim_{x\to -\infty} \frac{x^2+x-x^2}{\sqrt{x^2+x} - x} = \lim_{x\to -\infty} \frac{x}{|x|\sqrt{1+\frac{1}{x}} - x} =$$

$$= \lim_{x\to -\infty} \frac{x}{-x\sqrt{1+\frac{1}{x}} - x} = \lim_{x\to -\infty} \frac{1}{-\sqrt{1+\frac{1}{x}} - 1} = -\frac{1}{2}$$

Ombenfa (aug. 2020)

1. a) $\lim\limits_{x \to \ln 8} \dfrac{2e^{2x} - 17e^{x} + 8}{64 - e^{2x}} = \left| \begin{array}{l} t = e^{x} \\ x \to \ln 8 ,\ e^{\ln 8} = 8 \\ t \to 8 \end{array} \right| = \lim\limits_{t \to 8} \dfrac{2t^2 - 17t + 8}{64 - t^2}$

b) $\lim\limits_{x \to -\infty} \dfrac{\cos x}{x-1} = 0$

$-1 \leq \cos x \leq 1$

$-\dfrac{1}{x-1} \leq \dfrac{\cos x}{x-1} \leq \dfrac{1}{x-1}$

$\downarrow \qquad\qquad\qquad \downarrow$

$0 \qquad\qquad\qquad 0$

3. $x^2 + xy - y\ln(xy) = x - e^2$

$2x + y + xy' - y'\ln(xy) - y \cdot \frac{1}{xy}(y + xy') = 1$

(x_0, y_0), $k = f'(x_0, y_0)$

$y' = G(x, y)$

$y - y_0 = k(x - x_0)$

$y'(1, e^2) = \ldots$

$y - e^2 = y'(1, e^2)(x-1)$

4. $f(x) = ax^2 + \frac{4a^2}{x} + 5x$, $x = -2$

a) $f'(x) = 2ax - \frac{4a^2}{x^2} + 5$, $f'(-2) = -4a + a^2 + 5 = 0 \Rightarrow a_1 =$ $a_2 =$

$f''(x) = 2a + \frac{8a^2}{x^3}$, $f''(-2) = 2a - a^2 < 0$

6. $\sqrt{x+1} + a \quad , \ x = 3$

$2 + a = b$

$b = \lim_{x \to 3+} \frac{x^2 - 2x - 3}{x - 3} = \lim_{x \to 3+} \frac{(x-3)(x+1)}{x-3} = 4$

$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \Rightarrow \begin{matrix} b = 4 \\ 2 + a = b \end{matrix} \Rightarrow a = 2, b = 4$

7. 1 eller 3 olika lösningar men inte 2?

$P(x) = (x-1)^2 (x+5)$

$\downarrow \qquad \downarrow$

$x_{12} = 1 \qquad x_3 = -5$

$Q(x) = x^3$

$Q_1(x) = (x-2)^3$

2

$R(x) = (x+1)(x+2)(x+3)$

$-1 \quad -2 \quad -3$