**Övningar:** K1 : 28, 44, 45, 46

K2 : 1, 2, 3, 4, 5, 7

8, 9c, 10, 12, 13, 14

U28, sidan 53

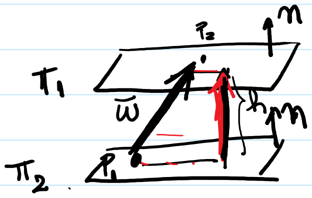
$\pi_1 : 2y + 4x + 8z = 28$

$\pi_2 : 2x + y + 4z + 3 = 0$

$d(\pi_1, \pi_2)$

$\pi_1 : \quad 4x + 2y + 8z = 28 \qquad \bar{n}_1 = (4, 2, 8) = 2\underline{(2, 1, 4)}$

$\pi_2 : \quad 2x + 1y + 4z = -3 \qquad \bar{n}_2 = (2, 1, 4)$




$\pi_1$ och $\pi_2$ är parallell.

$n_1 // n_2 \quad (n_1 = 2n_2)$

$$d(\pi_1, \pi_2) = h = \| \mathrm{proj}_n P_1P_2 \|$$

$P_1 \in \pi_1 \qquad P_1(0, 0, ?) \qquad 4(0) + 2(0) + 8z = 28$

$z = \frac{28}{8} = \frac{14}{4} = \frac{7}{2}$

$P_1(0, 0, 7/2)$

$P_2 \in \pi_2 \qquad P_2(0, 0, -3/4)$

$$\vec{w} = \overline{P_1P_2} = P_2 - P_1 = (0, 0, -3/4) - (0, 0, 14/4) = (0, 0, -17/4)$$

$$\mathrm{proj}_n P_1P_2 = \mathrm{proj}_n w = \frac{w \cdot n}{n \cdot n} n$$

$$h = \|\mathrm{proj}_{n}^{P_1P_2}\| = \left\| \frac{w\cdot n}{n\cdot n}\, n \right\| = \left| \frac{w\cdot n}{n\cdot n} \right| \|n\| = \frac{|w\cdot n|}{\|n\|^{\cancel{2}}} \cdot \cancel{\|n\|} =$$

$$= \frac{|w\cdot n|}{\|n\|} = \frac{|(0,0,-\frac{17}{4})\cdot(2,1,4)|}{\sqrt{2^2+1^2+4^2}} = \frac{|-17|}{\sqrt{4+1+16}} =$$

$$h = \frac{17}{\sqrt{21}} = \text{distans}(\Pi_1, \Pi_2)$$

---

U44 Sidan 55

Visa att $v, w, u$ ligger i ett plan

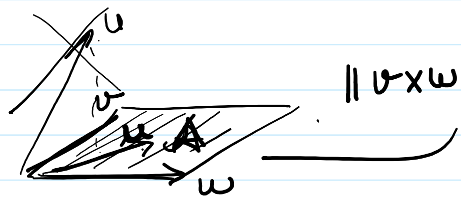
(använda vektorprodukt)

Svar:

$$v = \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix} \quad w = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} \quad u = \begin{pmatrix} -4 \\ 11 \\ -6 \end{pmatrix}$$

$$v \times w = \begin{vmatrix} i & j & k \\ 1 & 4 & -3 \\ 2 & -1 & 0 \end{vmatrix} = i(-3) - j(6) + k(-9)$$

$$v \times w = (-3,-6,-9)$$



$\|v \times w\| = \text{area } A$

$\|v \times w \cdot u\| = \text{volym} = 0$ Det betyder att $\{u, v, w\}$ ar i samma planet.

$$(-3,-6,-9)\cdot(-4,11,-6) =$$

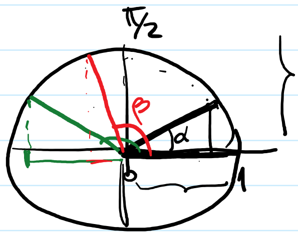
$$12 - 66 + 54 = 66 - 66 = \underline{\underline{0}}$$

Om det finns inge volym, betyder det att $u, v$ och $w$ är i samma planet. □

Kl, U 45 sidan 55

$v_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $v_2 = \begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix}$ $v_3 = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$

Mellan vilket par av dessa vektorer är vinkeln störst?




$\cos\alpha > 0$ när $0 < \alpha < \pi/2$

$\cos\beta < 0$ när $\pi/2 < \beta < \pi$

$u \cdot v = \|u\|\,\|v\| \cos\alpha \Rightarrow \cos\alpha = \dfrac{u \cdot v}{\|u\|\,\|v\|}$

$v_1 \cdot v_2 = \dfrac{(1,2,3)\cdot(-2,1,-2)}{\sqrt{1+4+9}\ \sqrt{4+1+4}} = \dfrac{-\cancel{2}+\cancel{2}-6}{\sqrt{14}\cdot\sqrt{9}} = \dfrac{-\cancel{6}}{\cancel{3}\sqrt{14}} = -\dfrac{2}{\sqrt{14}}$

$v_1 \cdot v_3 \Rightarrow \cos\gamma = 5/\sqrt{14}\sqrt{17}$ $\quad 0 < \gamma < \pi/2$

$v_2 \cdot v_3 \Rightarrow \cos\beta = -\dfrac{4}{\sqrt{17}}$

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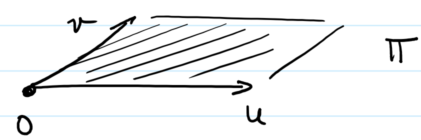
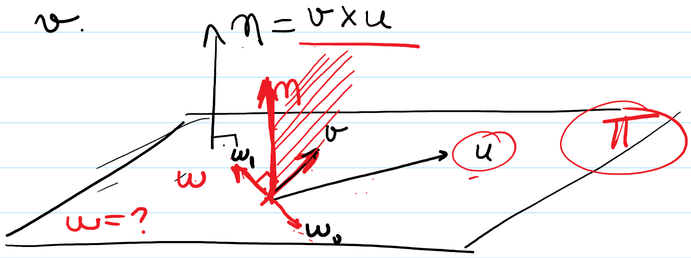
1.46 $\quad u = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $v = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$




Bestäm en vektor i detta plan ($\neq \vec{0}$) som är ortogonal mot $v$.




1°) $n = v \times u$

$n \perp v$ och $n \perp u$

2°) $\boxed{\vec{n} \times v = \vec{q}}$ $\quad q \perp n$ och $\perp v \Rightarrow q \in \pi$

1) $n = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 2 & -1 & 2 \end{vmatrix} = i\,(4+3) - j\,(2-6) + k\,(-1-4) =$

$i\,(7) - j\,(-4) + k\,(-5)$

$n = (7, 4, -5)$

2°) $w = n \times v = \begin{vmatrix} i & j & k \\ 7 & 4 & -5 \\ 2 & -1 & 2 \end{vmatrix}$

$= i\,[8 - 5] - j\,[14 + 10] + k\,[-7 - 8] =$

$= i\,(3) - j\,(24)\ \ k\,(-15)$

$\left.\begin{array}{ll} w_0 & (3, -24, -15) \\ w_1 & (-3, 24, 15) \end{array}\right\}$ är sant att $w \in \Pi$ ?

Vi visar att det är sant.

$\Pi$: $(x, y, z) \cdot (7, 4, -5) = 0 \Rightarrow 7x + 4y - 5z = 0$

$w \in \Pi \Rightarrow$ coordinates of $w$ satisfy the equation of $\Pi$.

$\underbrace{7(3)}_{21} + 4(-24) - 5(-15) =$

$21 \quad -96 \quad +75 \quad = -96 + 96 = 0$ ✓

We found one vector $w \perp v$ & $w \in \Pi$.

$\alpha w$ also satisfies the same restrictions. $\square$

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## Kapitel 2

Ex (1, 2, 3, 4, 5) Sidan 83.

2.1 $A = \begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix}$ $B = \begin{pmatrix} 2 & -2 \\ 4 & 1 \end{pmatrix}$ $A_{2\times2}$ $B_{2\times2}$

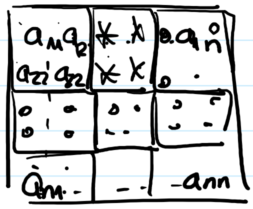
$C = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 4 & 0 \\ 7 & 2 & 5 \end{pmatrix}$ $D = \begin{pmatrix} -4 & 5 & 6 \\ -2 & 3 & 1 \\ 0 & 9 & 3 \end{pmatrix}$

a) $A + B$ ✓ $A_{2\times2} + B_{2\times2} = [a_{ij} + b_{ij}]$ $\begin{array}{l} i = 1,2 \\ j = 1\ 2 \end{array}$

a) $A+B$ ✓ $A_{2\times 2} + B_{2\times 2} = \left[a_{ij} + b_{ij}\right]$ $\quad i = 1,2$, $j = 1,2$

$B+C$ $\quad B_{2\times 2} + C_{3\times 3}$ ??? Ej definerad!

Anmärkning



$C+D \longrightarrow C+D = \left[c_{ij} + d_{ij}\right]$

$-3C \longrightarrow -3C = \left[-3\cdot c_{ij}\right]$

$2A+B \longrightarrow 2A+B = \left[2a_{ij}\right] + \left[b_{ij}\right] =$

$$\left[2a_{ij} + b_{ij}\right]$$

Man måste definera på olika sätt, när man har 2 matriser med olika dimensioner

---

K2.2 $\quad C = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 4 & 0 \\ 7 & 2 & 5 \end{pmatrix} \quad D = \begin{pmatrix} -4 & 5 & 6 \\ -2 & 3 & 1 \\ 0 & 9 & 3 \end{pmatrix}$

$u = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} \quad v = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \quad C\cdot D = \begin{pmatrix} -4 & 5-18 & 6-6 \\ 4 & -15+12 & -14 \end{pmatrix}$

a) Beräkna $Cu$ och $C(2u)$ $\qquad C(2u) = 2\,C(u)$

b) $D(u+v)$ och $Du + Dv$ $\qquad D(u+v) = Du + Dv$

c) $(C+D)v$ och $Cv + Dv$ $\qquad (C+D)v = Cv + Dv$

2.3 a) $AB \quad \begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} 2 & -2 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 1\cdot 2 - 5\cdot 4 & -2-5 \\ 8+12 & -8+3 \end{pmatrix} = \begin{pmatrix} -18 & -7 \\ 20 & -5 \end{pmatrix}$ $\neq$

$BA \quad \begin{pmatrix} 2 & -2 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 2-8 & -10-6 \\ 4+4 & -20+3 \end{pmatrix} = \begin{pmatrix} -6 & -16 \\ 8 & -17 \end{pmatrix}$

$A^2 = A\cdot A$

$\cancel{BC} \quad \underbrace{B_{2\times 2}\cdot C_{3\times 3}}_{??}$ ? Det funkar inte!

$CD \quad C_{3\times 3}\cdot D_{3\times 3}$

$DC \quad D_{3\times 3}\cdot C_{3\times 3}$ $\qquad$ in generell, $CD \neq DC$.

b) $A(A+B) = A^2 + A\cdot B$

c) $A(BA) = (AB)A$

Anmärkning $\boxed{ABA \neq A^2 \cdot B}$

2.4 a) $A^t$, $B^t$, $C^t$, $D^t$

$A = \begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix}$ $A^t = \begin{pmatrix} 1 & 4 \\ -5 & 3 \end{pmatrix}$

$B = \begin{pmatrix} 2 & -2 \\ 4 & 1 \end{pmatrix}$ $B^t = \begin{pmatrix} 2 & 4 \\ -2 & 1 \end{pmatrix}$

$C = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 4 & 0 \\ 7 & 2 & 5 \end{pmatrix}$ $C^t = \begin{pmatrix} 1 & -3 & 7 \\ 0 & 4 & 2 \\ -2 & 0 & 5 \end{pmatrix}$

$D = \begin{pmatrix} -4 & 5 & 6 \\ -2 & 3 & 1 \\ 0 & 9 & 3 \end{pmatrix}$ $D^t = \begin{pmatrix} -4 & -2 & 0 \\ 5 & 3 & 9 \\ 6 & 1 & 3 \end{pmatrix}$

A är symmetrisk om $A = A^t$

A, B, C, D är inte symmetrisk

⊛ $\underbrace{(AB)^t \quad A^t \cdot B^t \quad \text{och} \quad B^t A^t}$

Egenskap: $(AB)^t = B^t \cdot A^t$

$AB = \begin{pmatrix} -18 & -7 \\ 20 & -5 \end{pmatrix}$ $(AB)^t = \begin{pmatrix} -18 & 20 \\ -7 & -5 \end{pmatrix}$

$B^t = \begin{pmatrix} 2 & 4 \\ -2 & 1 \end{pmatrix}$ $A^t = \begin{pmatrix} 1 & 4 \\ -5 & 3 \end{pmatrix}$ $\Rightarrow \underset{B^t \cdot A^t}{} \begin{pmatrix} 2-20 & 8+12 \\ -2-5 & -8+3 \end{pmatrix} = \begin{pmatrix} -18 & 20 \\ -7 & -5 \end{pmatrix}$

$= -6 \neq$

$A^t \cdot B^t \quad \begin{pmatrix} 1 & 4 \\ -5 & 3 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ -2 & 1 \end{pmatrix} = \Big( 2-8$

$$\boxed{A^t . B^t \neq (AB)^t}$$

**2.5** $A = \begin{pmatrix} 1 & 4 \\ 3 & -5 \end{pmatrix}$ $B = \begin{pmatrix} 2 & -2 \\ 4 & 1 \end{pmatrix}$ $C = \begin{pmatrix} -4 & 5 & 6 \\ -2 & 3 & 1 \\ 0 & 9 & 3 \end{pmatrix}$

$\det A = (-5) - (12) = -17$

$\det B = (2) - (-8) = 2+8 = 10$

$$C = (-4)(3)(3) + (-2)(9)(6) + (0)\cancel{(5)}(1)$$

$$- \left( \cancel{0.3.6} + 9.1.(-4) + 3(-2)(5) \right)$$

$$= \cancel{-36} - 108$$
$$- \left( \cancel{-36} - 30 \right)$$

$$\begin{array}{r} -108 \\ +\ 30 \\ \hline 78 \end{array} \qquad -78$$

$$C = \begin{array}{ccc} 1+1 & 1+2 & 1+3 \\ \end{array}$$
$$\begin{pmatrix} -4 & 5 & 6 \\ -2 & 3 & 1 \\ 0 & 9 & 3 \end{pmatrix} = (-1)^2(-4) \det \begin{pmatrix} 3 & 1 \\ 9 & 3 \end{pmatrix} + (-1)^3 (5) \det \begin{pmatrix} -2 & 1 \\ 0 & 3 \end{pmatrix}$$

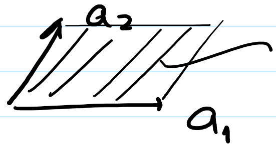
$$+ (-1)^{1+3} (6) \det \begin{pmatrix} -2 & 3 \\ 0 & 9 \end{pmatrix}$$

$$= (-4) \cancel{(9-9)} - 5(-6) + 6(-18)$$

$$= +30 - 108 = -78$$

$$\begin{array}{r} 4 \\ 18 \\ \times\ 6 \\ \hline 108 \end{array}$$

b) $A = \begin{pmatrix} | & | \\ a_1 & a_2 \\ | & | \end{pmatrix}$

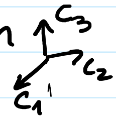



Area $= |\det A|$

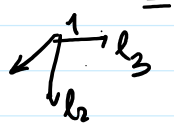
$B = \begin{pmatrix} | & | \\ b_1 & b_2 \\ | & | \end{pmatrix}$


Area $= |\det B|$

$C = \begin{pmatrix} c_1 & c_2 & c_3 \end{pmatrix}$ volym $= |\det C|$

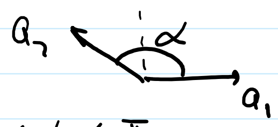
$C = \begin{pmatrix} c_1 & c_2 & c_3 \end{pmatrix}$ volym  $= |\det C|$



$C^t = \begin{pmatrix} — c_1 — \\ — c_2 — \\ — c_3 — \end{pmatrix}$ volym$_\ell$  $= |\det C^t|$

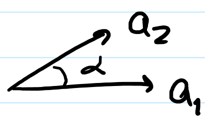


**2e:** Viktig! $A = \begin{pmatrix} a_1 & a_2 \end{pmatrix}$ $\alpha = \angle(a_1, a_2)$





eller

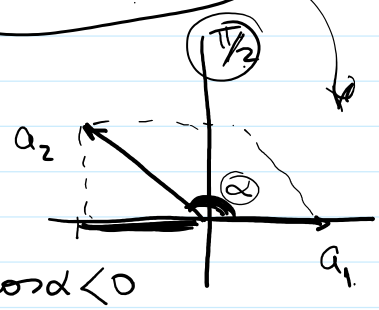




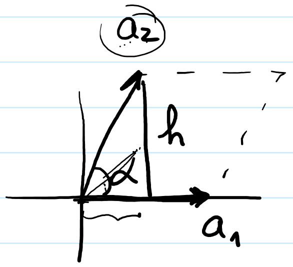
$0 < \alpha < \pi/2$

$\pi/2 < \alpha < \pi$

$\det A = -17$









Area $= \|a_1\| \cdot h$

$\|a_1\| \, \underbrace{\|a_2\| \sin\alpha}$

$\cos\alpha < 0$

Om $\pi/2 < \alpha < \pi$

$a_1 \cdot a_2 = \|a_1\| \|a_2\| \cos\alpha$

$\dfrac{a_1 \cdot a_2}{\|a_1\| \cdot \|a_2\|} = \cos\alpha$

$a_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ $a_2 = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

$a_1 \cdot a_2 = 4 - 3(5) = 4 - 15 = \underline{-11}$

$\cos\alpha = \dfrac{-11}{\sqrt{1+9}\sqrt{16+25}} = \dfrac{-11}{\sqrt{10}\sqrt{41}} = \dfrac{-11}{\sqrt{410}}$

$\alpha = \arccos\left(-\dfrac{11}{\sqrt{410}}\right)$

**2f** Vad har $(c_1\ c_2\ c_3)$ för orientering?

När $\det C > 0$, betyder det att orientering är $(+)$

Högerhand

Höger hand

När $\det C < 0$, betyder det att orientering är (-)
Vänster hand

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## 2.8 Inverse Matris

$$\underset{2\times 2}{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \longrightarrow A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

a) $A, B$ är inverterbar (Det finns $A^{-1}$ och $B^{-1}$)

Lös: $AXB + C = D$ (? $X = ?$)

Svar:

$$AXB + C = D$$

$$AXB = (D - C)$$

$$A^{-1}(AXB) = A^{-1}(D-C)$$

$$\underbrace{(A^{-1}A)}_{Id} X B = A^{-1}(D-C)$$

$$XBB^{-1} = A^{-1}(D-C)B^{-1}$$

$$\boxed{X = A^{-1}(D-C)B^{-1}}$$

2.8 b) $I_n + B$ är inverterbar $(I_n+B)^{-1}$

$$AXB + C = D - AX$$

$$AXB + AX = D - C$$

$$AX(B+I_n) = D - C$$

$$X\underbrace{(B+I_n)}_{\text{inverterbar}} = (D-C)A^{-1}$$

$$\boxed{X = (I_n+B)^{-1} \cdot (D-C)A^{-1}}$$

lösning.