Ö_4_Feb_15e

den 15 februari 2021 09:53

Basupp $\begin{cases} \text{Kap 3: } 1, 2, 3, 5, 6, 8, 9, 11, 12, 14 \\ \text{Kap 4: } 1, 2, 3 \end{cases}$

(1) $B = \begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix}$ $\quad u = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ $\quad v = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ $\quad V = \mathbb{R}^2$

$$f_B(u) = B \cdot u = \underset{2\times 2}{\begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix}} \underset{2\times 1}{\begin{pmatrix} 1 \\ 5 \end{pmatrix}} = \begin{pmatrix} 1.1 - 5.5 \\ 4.1 + 3.5 \end{pmatrix} = \begin{pmatrix} 1-25 \\ 4+15 \end{pmatrix} = \begin{pmatrix} -24 \\ 19 \end{pmatrix}$$

$$f_B(v) = B.v = \begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -3-5 \\ -12+3 \end{pmatrix} = \begin{pmatrix} -8 \\ -9 \end{pmatrix}$$

f linjär $\boxed{f(u+v) = f(u) + f(v)}$

$$f_B(u+v) = B(u+v) = \begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix}\overbrace{\begin{pmatrix} 1-3 \\ 5+1 \end{pmatrix}}^{u+v}$$

$$\begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} -2 \\ 6 \end{pmatrix} = \begin{pmatrix} -2-30 \\ -8+18 \end{pmatrix} = \begin{pmatrix} -32 \\ +10 \end{pmatrix}$$

$$f_B(u) + f_B(v) = \begin{pmatrix} -24 \\ 19 \end{pmatrix} + \begin{pmatrix} -8 \\ -9 \end{pmatrix} = \begin{pmatrix} -32 \\ 10 \end{pmatrix}$$

$$f(\alpha u) = \alpha f(u) \qquad \alpha \in \mathbb{R}$$

$$\alpha = 2 \Rightarrow 2u = 2\begin{pmatrix} 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 10 \end{pmatrix}$$

$$f(2u) = B\begin{pmatrix} 2 \\ 10 \end{pmatrix} = \begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ 10 \end{pmatrix} = \begin{pmatrix} 2-50 \\ 8+30 \end{pmatrix} = \begin{pmatrix} -48 \\ 38 \end{pmatrix}$$

$$2f(u) = 2\begin{pmatrix} -24 \\ 19 \end{pmatrix} = \begin{pmatrix} -48 \\ 38 \end{pmatrix}$$

2) Samma ide.

3) Sidan 113. f = linjär avbildning i $V = \mathbb{R}^3$ rummet

$$f(e_x) = \begin{pmatrix} 1 \\ 6 \\ 7 \end{pmatrix} \quad f(e_y) = \begin{pmatrix} 2 \\ 8 \\ 9 \end{pmatrix} \quad f(e_z) = \begin{pmatrix} 4 \\ 5 \\ 8 \end{pmatrix}$$

? matrisen A så att $f(u) = A \cdot u$?

Lösning $B = \{e_x, e_y, e_z\}$

$$u = \alpha e_x + \beta e_y + \gamma e_z =$$

$$u = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

$$f(u) = f(\alpha e_x + \beta e_y + \gamma e_z) \overset{f \text{ linjär}}{=}$$

$$\alpha f(e_x) + \beta f(e_y) + \gamma f(e_z) =$$

$$\underbrace{\begin{bmatrix} | & | & | \\ f(e_x) & f(e_y) & f(e_z) \\ | & | & | \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}}_{[u]}$$

$[u]$ med koordinater på bas $B = \{e_x\, e_y\, e_z\}$

35 f är linjär $V = \mathbb{R}^3$

$$f\left(\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \quad f\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 2 \\ 8 \\ 9 \end{pmatrix} \quad f\left(\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$$

Bestäm matrisen A så att $f = f_A \Leftrightarrow f(u) = A \cdot u$.

$$A = \begin{pmatrix} f(e_x) & f(e_y) & f(e_z) \end{pmatrix}$$

$$f(u) = \begin{pmatrix} f(e_x) & f(e_y) & f(e_z) \end{pmatrix} \begin{pmatrix} x \\ \vdots \end{pmatrix}$$

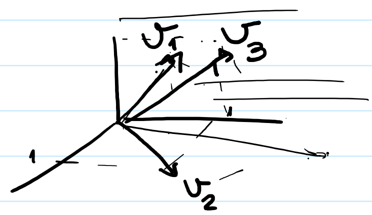
$$f_A(u) = \left(f(e_x)\ f(e_y)\ f(e_z) \right) \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{u = xe_x + ye_y + ze_z}$$

$\underline{1^{st}\ \text{try}}$: $v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $v_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ $v_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

Om $D = \{v_1, v_2, v_3\}$ är också bas för $\mathbb{R}^3$

då kunde $\boxed{\vec{u} \in \mathbb{R}^3}$

$$\bar{u} = \alpha v_1 + \beta v_2 + \gamma v_3$$

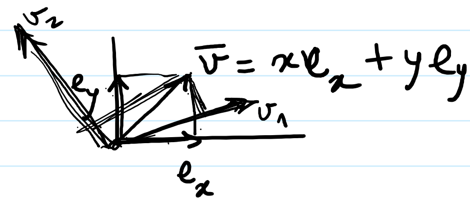


$$f(u) = f(\alpha v_1 + \beta v_2 + \gamma v_3) =$$
$$= \alpha f(v_1) + \beta f(v_2) + \gamma f(v_3)$$
$$= \underbrace{\left[f(v_1)\ f(v_2)\ f(v_3) \right]}_{A} \underbrace{\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}}_{u = (\alpha, \beta, \gamma)_D}$$

D Dsom Bas.

$u = (x, y, z) = x e_x + y e_y + z e_z$ standard $\underline{\text{bas}}$

$$\begin{array}{c} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \\ v_1 \quad v_2 \quad v_3 \end{array} \underbrace{\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}}_{} = \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{u_{\text{Bas } B}}$$



$$\left[\begin{array}{ccc|c} \boxed{1} & 1 & 0 & x \\ 1 & 1 & 1 & y \\ 1 & 0 & 1 & z \end{array}\right] \begin{array}{l} \\ R_2 \leftarrow R_2 - R_1 \\ R_3 \leftarrow R_3 - R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & x \\ 0 & 0 & 1 & y-x \\ 0 & -1 & 1 & z-x \end{array}\right] \begin{array}{l} \\ \\ R_3 \leftarrow (-1) R_3 \end{array} \quad R_2 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & x \\ 0 & 1 & -1 & x-z \\ 0 & 0 & 1 & y-x \end{array}\right] \quad R_2 \leftarrow R_2 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & x \\ 0 & 1 & 0 & y-z \\ 0 & 0 & 1 & y-x \end{array}\right] \quad R_1 \leftarrow R_1 - R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & x-y+z \\ 0 & 1 & 0 & y-z \\ 0 & 0 & 1 & y-x \end{array}\right]$$

$$\begin{aligned} 1\alpha + 0\beta + 0\gamma &= x-y+z \\ 0\alpha + 1\beta + 0\gamma &= y-z \\ 0\alpha + 0\beta + 1\gamma &= y-x \end{aligned} \qquad \begin{aligned} \alpha &= x-y+z \\ \beta &= y-z \\ \gamma &= y-x \end{aligned}$$

$$f(u)_D = \begin{bmatrix} \underbrace{\begin{matrix} 2 \\ 3 \\ 4 \end{matrix}}_{f(v_1)} & \underbrace{\begin{matrix} 2 \\ 8 \\ 9 \end{matrix}}_{f(v_2)} & \underbrace{\begin{matrix} 1 \\ 5 \\ 2 \end{matrix}}_{f(v_3)} \end{bmatrix} \underbrace{\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}}_{(u)_D = \alpha v_1 + \beta v_2 + \gamma v_3}$$

$$\begin{aligned} \alpha &= x - y + z \\ \beta &= 0 + 1y - 1z \\ \gamma &= -1x + 1y + 0z \end{aligned} = \underbrace{\begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ -1 & 1 & 0 \end{pmatrix}}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$f(u)_{B\ \text{standard}} = \underbrace{\begin{bmatrix} 2 & 2 & 1 \\ 3 & 8 & 5 \\ 4 & 9 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ -1 & 1 & 0 \end{bmatrix}} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{(u)_B = xe_x + ye_y + ze_z}$$

$$\underbrace{\begin{bmatrix} 1 & 1 & 0 \\ -2 & 10 & -5 \\ 2 & 7 & -5 \end{bmatrix}}_{(A)}$$

Standard Base

Test

Test

Standard Base

$$f_A\begin{pmatrix}x\\y\\z\end{pmatrix} = \underbrace{\begin{bmatrix}1 & 1 & 0\\ -2 & 10 & -5\\ 2 & 7 & -5\end{bmatrix}}_{\text{A matrisen som är relaterad till f när B = standard Bas}}\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}1+1+0\\ -2+10-5\\ 2+7-5\end{bmatrix} = \begin{bmatrix}2\\3\\4\end{bmatrix} \checkmark$$

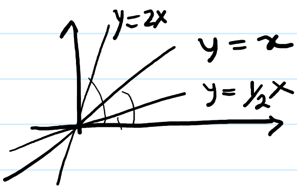
A matrisen som är relaterad till f när B = standard Bas

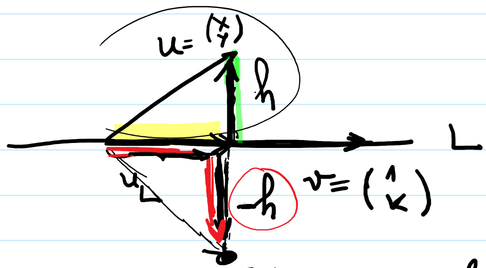
3.6 Bestäm matrisen för den linjära avbildningen i planet ($V=\mathbb{R}^2$) som ges av spegling i linjen $y=kx$, där $k\in\mathbb{R}$.

Lösning. $f: V=\mathbb{R}^2 \longrightarrow V=\mathbb{R}^2$

$u=\begin{pmatrix}x\\y\end{pmatrix} \longrightarrow f(u) = S.u = S\begin{pmatrix}x\\y\end{pmatrix}$ S = matrisen av spegling.

Linje: $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x\\kx\end{pmatrix} = \alpha\begin{pmatrix}1\\k\end{pmatrix}$ $\alpha\in\mathbb{R}$ K = parameter som påverkar vinkel.





$u = u_L + h$

$h = u - u_L$

$-h = -u + u_L = u_L - u$

$S(u) = u_L + (-h)$

$= u_L + u_L - u$

$$\boxed{S(u) = 2u_L - u}$$

$v = \begin{pmatrix}1\\k\end{pmatrix}$ $k\in\mathbb{R}$. fix

$u = (x,y)$

$v\cdot v = \|v\|^2 = (\sqrt{1+k^2})^2$
$= (1+k^2)$

$$u_L = \mathrm{proj}_v u = cv = \frac{u\cdot v}{v\cdot v}v$$

$$u_L = \frac{\begin{pmatrix} x \\ y \end{pmatrix} \cdot \begin{pmatrix} 1 \\ k \end{pmatrix}}{(1+k^2)} \begin{pmatrix} 1 \\ k \end{pmatrix}$$

$$u_L = \frac{(x+ky)}{(1+k^2)} \begin{pmatrix} 1 \\ k \end{pmatrix}$$

$$S(u) = S\begin{pmatrix} x \\ y \end{pmatrix} = 2\,\frac{(x+ky)}{(1+k^2)} \begin{pmatrix} 1 \\ k \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} =$$

$$\left(\frac{2(x+ky)}{(1+k^2)}\,1 - x \,,\; \frac{2(x+ky)}{(1+k^2)}\,k - y \right) =$$

$$\left(\frac{2x + 2ky - x(1+k^2)}{(1+k^2)} \,,\; \frac{2xk + 2k^2y - y(1+k^2)}{(1+k^2)} \right) =$$

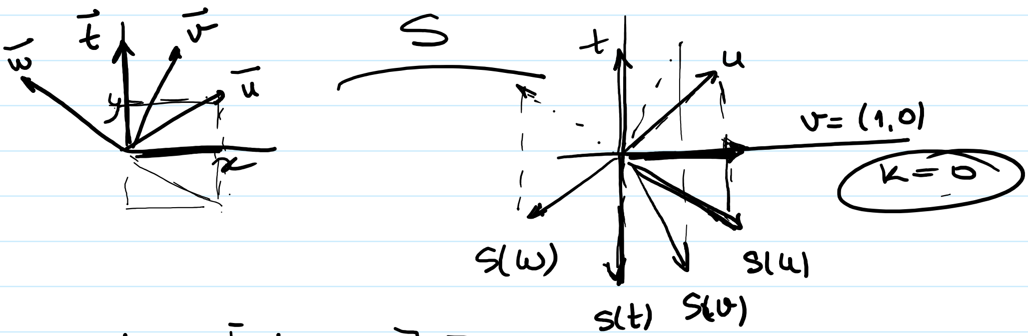
$$\frac{1}{(1+k^2)} \left(x(1-k^2) + 2ky \,,\; 2xk + \cancel{2k^2y} - y - \cancel{yk^2} \right)$$

$$\frac{1}{(1+k^2)} \left(x(1-k^2) + 2ky \,,\; \boxed{2kx + (k^2-1)y} \right)$$

$$\frac{1}{(1+k^2)} \underbrace{\begin{bmatrix} (1-k^2) & 2k \\ 2k & -(1-k^2) \end{bmatrix}}_{} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{(1+k^2)} \begin{bmatrix} (1-k)^2x + 2ky \\ 2kx - (1-k^2)y \end{bmatrix}$$

$A = S$ speglingen matris.

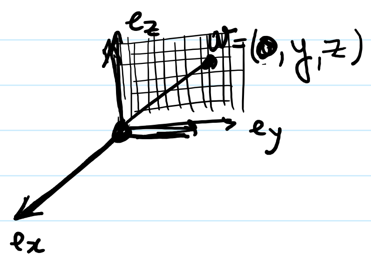
Definitionsmängd

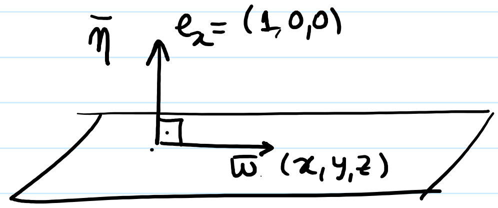


$$k=0 \quad S\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{(1+0^2)} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -y \end{bmatrix}$$

$\bar{\eta} \wedge e_2 = (1,0,0)$

3.8 $V = \mathbb{R}^3$





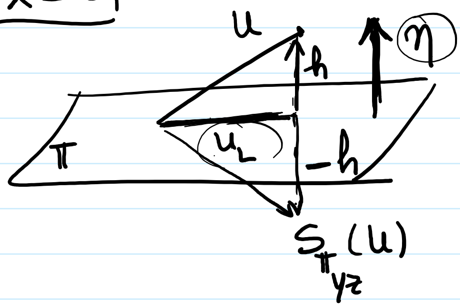
$\underbrace{\pi_{yz}\text{ - planet}}$

$\bar{\eta} \cdot (x, y, z) = \underline{0} \Rightarrow$

$(1,0,0) \cdot (x,y,z) = \underline{1}x + 0 + 0 = 0 \Rightarrow$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $\left\{ \begin{array}{l} \boxed{x = 0} \\ y = \text{fri} \\ z = \text{fri} \end{array} \right.$

$\boxed{X = 0}$



$\underbrace{h = \text{proj}_\eta u = u_\eta}$

$u = u_L + h$

$u - h = u_L$

$\downarrow$

$S_\pi(u) = u_L - h$

$= u - h - h$

$$\boxed{S_\pi(u) = u - 2h}$$

$= \begin{pmatrix} \\ y \\ z \end{pmatrix} \quad \begin{matrix} x \\ 0 \\ 0 \end{matrix} \Big)$

$= \begin{pmatrix} x - 2x \\ y - 0 \\ z - 0 \end{pmatrix} =$

$= \begin{pmatrix} -x \\ y \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{S_{\pi_{yz}}} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$u = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \bar{\eta} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$h = u_\eta = \dfrac{u \cdot \eta}{\eta \cdot \eta}\, \eta =$

$\dfrac{\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}{1} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} =$

$x \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

Avs 3.5 Upp. $\underline{3.9}$ Sida 114

$V = \mathbb{R}^2$

$\mathbb{R}^2 \quad \mathbb{R}^2$

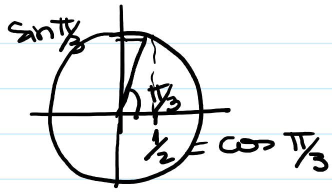
Vad är matrisen $f: V \longrightarrow V$

$u = \binom{x}{y} \longrightarrow f(u) = \underbrace{A}_{?}\binom{x}{y}$

$A = \boxed{S_L} \circ \boxed{R_{\pi/3}}$

$L: y = x$

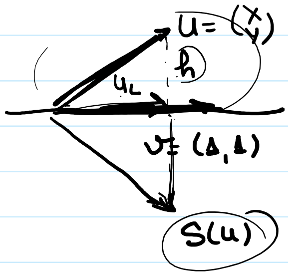
1) $R_{\pi/3}$



$180 \,|\underline{3}$
$\quad 60°$

$$R_{\pi/3} = \begin{pmatrix} \cos \pi/3 & -\sin \pi/3 \\ \sin \pi/3 & \cos \pi/3 \end{pmatrix} = \begin{pmatrix} & -\frac{\sqrt{3}}{2} \\ & \frac{1}{2} \end{pmatrix}$$

2) S_L



$L: y = x \qquad v = (x, x) = x\underbrace{(1,1)}_{v} \quad x \in \mathbb{R}$

$u = u_L + h \qquad \boxed{h = u - u_L}$

$$\boxed{S(u) = u_L - h} = u_L - (u - u_L) = \boxed{2u_L - u}$$

$$u_L = \frac{u \cdot v}{v \cdot v} v = \frac{(x,y)\cdot(1,1)}{(1,1)(1,1)}(1,1) =$$

$$= \frac{(x+y)}{(1+1)}(1,1) = \left(\frac{x+y}{2}\right)(1,1)$$

$$S(u) = 2u_L - u$$

$$\cancel{2}\left(\frac{x+y}{\cancel{2}}\right)(1,1) - (x,y)$$

$$\begin{pmatrix} x+y \\ x+y \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$$

$$\underbrace{\qquad\qquad}_{S(u)}$$

$$f(u) = f\begin{pmatrix}x\\y\end{pmatrix} = (S\circ R)\begin{pmatrix}x\\y\end{pmatrix} =$$

$$\begin{pmatrix}0&1\\1&0\end{pmatrix}\underbrace{\begin{pmatrix}1/2&-\sqrt{3}/2\\ \sqrt{3}/2&1/2\end{pmatrix}}_{A}\begin{pmatrix}x\\y\end{pmatrix}$$

$$\begin{pmatrix}\sqrt{3}/2&1/2\\ 1/2&-\sqrt{3}/2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix}\sqrt{3}&1\\ 1&-\sqrt{3}\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$$

Avsnitt 3.6 sidan 114

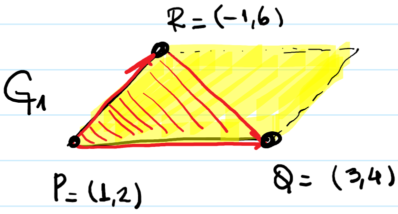
3.12 Låt $P=(1,2)$ $Q=(3,4)$ $R=(-1,6)$

a) Vad är arean av triangeln ΔPQR?

b) Låt $f:\mathbb{R}^2\to\mathbb{R}^2$ den linjära avbildning med matrisen $A=\begin{pmatrix}2&3\\8&2\end{pmatrix}$

Vad är arean av bilden $f(\Delta PQR)$ av triangeln ΔPQR.

Lösning (titta sidan 106)



$$PQ = Q - R = \begin{pmatrix}3\\4\end{pmatrix}-\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}2\\2\end{pmatrix}$$

$$PR = R - P = \begin{pmatrix}-1\\6\end{pmatrix}-\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}-2\\4\end{pmatrix}$$

$$\text{Arean } \Delta PQR = \frac{1}{2}\,\text{Arean}_{PQ}^{PR} = \frac{1}{2}\,\text{Arean}(G_1)$$

G_1 = parallellepiped med sidor $\overrightarrow{PQ}$, och $\nearrow PR$

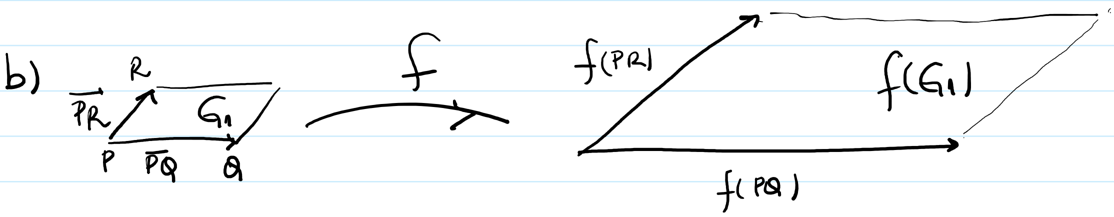
$$\text{Arean}_{\square} = \begin{cases} \|PQ\|\,\|PR\|\sin\alpha & \alpha\ \measuredangle(PQ, PR) \\ \text{eller} \\ \det\begin{bmatrix} PQ & PR \end{bmatrix} = \det\begin{bmatrix}2&-2\\2&4\end{bmatrix} = 8-(-4) = 12 \end{cases}$$

Då

$$\text{Arean}_{\Delta} = \frac{1}{2}\,\text{Arean}_{\square} = \frac{1}{2}(12) = 6$$

$$\text{Arean}_{\triangle} = \frac{1}{2}\,\text{Arean}_{\square} = \frac{1}{2}(12) = 6$$

b)



$$G = f(G_1)$$

$$\text{Arean } G = \text{Arean } f(G_1) = \det\left[\, f(\overline{PQ}) \quad f(\overline{PR}) \,\right]$$

$$= \det\left(\left[A\cdot\overline{PQ} \quad A\cdot\overline{PR}\right]\right) = \det\left(A\cdot\left[\overline{PQ} \quad \overline{PR}\right]\right)$$

$$= \det(A)\cdot\det\left(\overline{PQ} \quad \overline{PR}\right)$$

Då

$$\text{Arean } f(G_1) = \underbrace{\det\begin{pmatrix} 2 & 3 \\ 8 & 2 \end{pmatrix}}_{\det A \,=\, \underbrace{|4-24|}_{20}} \cdot \underbrace{\det\begin{pmatrix} 2 & -2 \\ 2 & 4 \end{pmatrix}}_{(\underbrace{8-(-4)}_{=\,(8+4)})\,=\,12} = 20\times 12 = 240$$

Men

$$\text{Arean } f(\Delta PQR) = \frac{1}{2}\,\text{Arean}_{\square}\, f(G_1)$$

$$= \frac{1}{2}\,240 = 120.$$

3.13 $V = \mathbb{R}^3$, K = klot med radien r

a) Vad är volym av K?

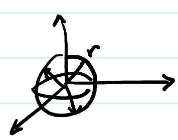
b) Låt f vara den linjära avbildning med matrisen

$$A = \begin{pmatrix} -4 & 5 & 6 \\ -2 & 3 & 1 \\ 0 & 9 & 3 \end{pmatrix}.$$

Vad är volymen av bilden $f(K)$ av K?

Lösning:

a) K = klot med radien r



$\bar{v} \in K \quad \bar{v} = \alpha e_1 + \beta e_2 + \gamma e_3$

$$\text{Volym av } K = \frac{4}{3}\pi r^3$$

b) Sats 3.31

$\left[\begin{array}{l} D = \text{område i rummet} \\ D' = \text{bilden av } D \text{ under } f_A \text{, } A \text{ matris} \end{array}\right.$ $\qquad D = K$, $\quad D' = f_A(K)$

$\left[\text{Då volym}(D') = |\det(A)| \cdot \text{volym}(D)\right.$

Då $\text{volym}(f_A(K)) = \left|\det\begin{pmatrix} -4 & 5 & 6 \\ -2 & 3 & 1 \\ 0 & 9 & 3 \end{pmatrix}\right| \cdot \frac{4}{3}\pi r^3$

$|\det A| = |(-4)(3)(3) + (-2)(9)(6) + 0.5.3$

$-(0 - 36 - 30)| =$

$= |\cancel{-36} - 18.6 \cancel{+36} + 6.5| = |6(-18+5)| = |6(-13)| =$

$= |-78| = +78$

Då $\text{volym}(f_A(K)) = \cancel{78}.\frac{4}{\cancel{3}}\pi . r^3 = 26 \cdot 4 \pi r^3 = \underline{104\pi r^3}$

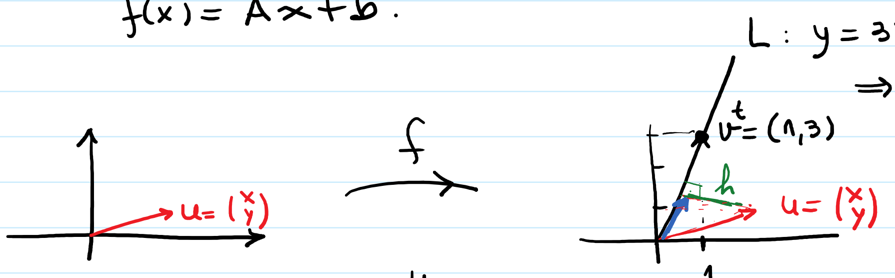
Avsnitt 3.7 sidan 115

3.14 a) Bestäm matrisen för ortogonal projektion på linje $L: y = 3x$

b) Bestäm en matris A och en vektor b så att den affina avbildning f som är projektion av punkten i planet på linjen $y = 3x+1$ ges av $f(x) = Ax + b$.

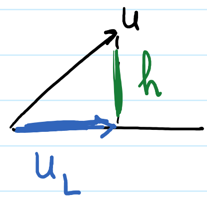
Lösning

a)



$\Rightarrow v = \binom{x}{3x} = x\binom{1}{3}$

$\boxed{v = \binom{1}{3}}$



$u = u_L + h$ $\left\{\begin{array}{l} \text{med } h \perp v \\ \& \\ u_L = cv \end{array}\right.$

$u_L = \frac{\binom{x}{y}\cdot\binom{1}{3}}{\binom{1}{3}\cdot\binom{1}{3}}\binom{1}{3}$

$$\overline{\begin{pmatrix}1\\3\end{pmatrix}\cdot\begin{pmatrix}1\\3\end{pmatrix}} \quad 3)$$

$$u_L = \left(\frac{x\cdot 1 + y\cdot 3}{1+9}\right)\begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}\frac{1}{10}x + \frac{3}{10}y \\ \frac{3x}{10} + \frac{3.3y}{10}\end{pmatrix} = \begin{pmatrix}1/10 & 3/10\\ 3/10 & 9/10\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$$

Då $A = \begin{pmatrix}1/10 & 3/10\\ 3/10 & 9/10\end{pmatrix} = 1/10\begin{pmatrix}1 & 3\\ 3 & 9\end{pmatrix}$

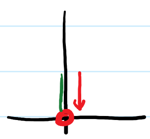
Anmärkning: Här $y = 3x$ är en Linje som origo tillhör till linjen.

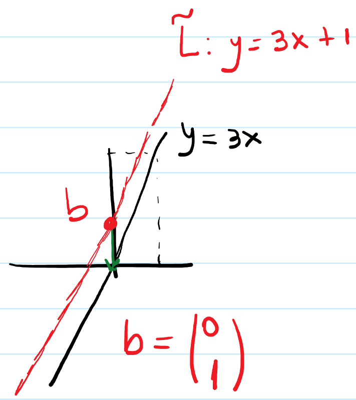
b) $f: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$u \longrightarrow f(u) = u_L \qquad u_L = proj_{\tilde{L}}\, u \qquad \tilde{L}: y = 3x + 1$

$\tilde{L} /\!/ L$ men Origo $(0,0) \notin \tilde{L}$
origo inte tillhör till $\tilde{L}$.

So nu $f(u) = (t_b \circ P_L \circ t_{-b})(u)$





1^{st}: $t_{-b}(u) = u - b$

$t_{-b}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix} - \begin{pmatrix}0\\1\end{pmatrix}$

$t_{-b}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x-0\\y-1\end{pmatrix}$

2^{nd}: från (a) har vi $A = \frac{1}{10}\begin{pmatrix}1 & 3\\ 3 & 9\end{pmatrix}$ som är projektion på linjen $L: y = 3x$.

Då $f\begin{pmatrix}x\\y\end{pmatrix} = t_b \circ \left(A\begin{pmatrix}x-0\\y-1\end{pmatrix}\right) =$

$A\begin{pmatrix}x-0\\y-1\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}$

$A\begin{pmatrix}x\\y\end{pmatrix} - A\begin{pmatrix}0\\1\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}$

$\uparrow$

$$\underset{\substack{\uparrow \\ \text{matris}}}{A}\begin{pmatrix}x\\y\end{pmatrix} \underbrace{- A\begin{pmatrix}0\\1\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}}_{\text{translation}}$$

$$A\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1/10 & 3/10\\ 3/10 & 9/10\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}3/10\\ 9/10\end{pmatrix}$$

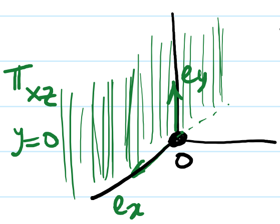
Då : $A\begin{pmatrix}x\\y\end{pmatrix} - \begin{pmatrix}3/10\\ 9/10\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix} =$

$$A\cdot\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}-3/10 + 0\\ -9/10 + 1\end{pmatrix} =$$

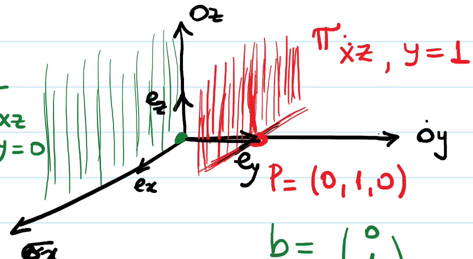
$$\boxed{f\begin{pmatrix}x\\y\end{pmatrix} = A\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}-3/10\\ 1/10\end{pmatrix} \quad \text{med } A = \begin{pmatrix}1/10 & 3/10\\ 3/10 & 9/10\end{pmatrix}} \qquad B = \begin{pmatrix}-3/10\\ 1/10\end{pmatrix}$$

3.15 Bestäm A och b så att den affina avbildning f som är speglingen av punkten i rummet i planet som ges av $y = 1$ ges av $f(x) = Ax + b$

Lösning.

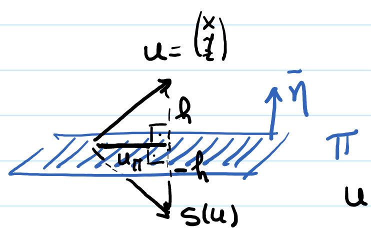


$$\underbrace{t_{-b}(u)}\qquad t_{-b}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}x\\y\\z\end{pmatrix} - \begin{pmatrix}0\\1\\0\end{pmatrix}$$



$b = \begin{pmatrix}0\\1\\0\end{pmatrix}$

Speglingen i planet Π



$u = u_\Pi + h$

$$h = \operatorname{proj}_\eta u = \frac{\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}a\\b\\c\end{pmatrix}}{\begin{pmatrix}a\\b\\c\end{pmatrix}\cdot\begin{pmatrix}a\\b\\c\end{pmatrix}}\begin{pmatrix}a\\b\\c\end{pmatrix}$$

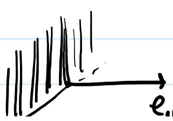
$u_\Pi = u - h \Rightarrow$

$s(u) = u_\Pi - h$

$s(u) = (u - h) - h$

$$\boxed{s(u) = u - 2h}$$

$\eta = \begin{pmatrix}a\\b\\c\end{pmatrix}$ $\quad \Pi_{xz}$



$e_{..} = \begin{pmatrix}0\\1\\0\end{pmatrix} = \eta$ av $\Pi : y = 0$

$\eta = \begin{pmatrix} a\\ b\\ c\end{pmatrix}$ Π_{xz}, $y=0$

$e_y = \begin{pmatrix}0\\1\\0\end{pmatrix} = \eta$ av $\Pi : y=0$

Då $h = \dfrac{\begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}0\\1\\0\end{pmatrix}}{1}\begin{pmatrix}0\\1\\0\end{pmatrix} = y\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\y\\0\end{pmatrix}$

Och $S(u) = \begin{pmatrix}x\\y\\z\end{pmatrix} - 2\begin{pmatrix}0\\y\\0\end{pmatrix} = \begin{pmatrix}x-0\\y-2y\\z-0\end{pmatrix} = \begin{pmatrix}x\\-y\\z\end{pmatrix}$

$$S(u) = \begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = S_{3\times3}\cdot u_{3\times1}$$

Slutligen att skappa f bör vi

$$f(u) = (t_b \circ S \circ t_{-b})(u) =$$

$$t_b \circ S\left(\begin{pmatrix}x\\y\\z\end{pmatrix} - \begin{pmatrix}0\\1\\0\end{pmatrix}\right) =$$

$$t_b\left(\underbrace{\begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{pmatrix}}\begin{pmatrix}x\\y\\z\end{pmatrix} - \begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}\right) = \underbrace{u}_{\circledast} + \underbrace{b}_{\boxed{*}}$$

$$\underbrace{S.\begin{pmatrix}x\\y\\z\end{pmatrix} - S\begin{pmatrix}0\\1\\0\end{pmatrix}}_{*} + \underbrace{\begin{pmatrix}0\\1\\0\end{pmatrix}}_{\boxed{*}} =$$

$$\underbrace{\begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{pmatrix}}\begin{pmatrix}x\\y\\z\end{pmatrix} + \begin{pmatrix}0\\1\\0\end{pmatrix} + \begin{pmatrix}0\\1\\0\end{pmatrix} =$$

$$\boxed{Ax + \underbrace{\begin{pmatrix}0\\2\\0\end{pmatrix}}_{b}} \quad \boxtimes$$

3.18 $L :// \overrightarrow{ox}$ L går genom $P = (0,1)$

P i planet, $P_s = \text{spegling}_L(P)$

Till exempel: $Q = (2,2)$ $Q_s = (2,0)$

$f(\overline{OP}) = \overline{OP_s}$ $\qquad f\begin{pmatrix}2\\2\end{pmatrix} = \begin{pmatrix}2\\0\end{pmatrix}$

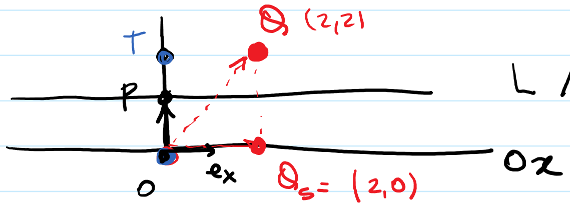
Visa att detta är inte en linjär avbildning.

Lösning

$P = (0,1)$

$\overline{OP} = \begin{pmatrix}0\\1\end{pmatrix}$

Spegling i linjen L.



$f\begin{pmatrix}0\\2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

$f\begin{pmatrix}2\\2\end{pmatrix} = \begin{pmatrix}2\\0\end{pmatrix}$

$f\begin{pmatrix}0\\2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$ och $f\begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}0\\2\end{pmatrix}$

Men $\begin{bmatrix} f: V \to V \\ f \text{ linjär} \end{bmatrix} \Rightarrow \begin{bmatrix} f(0) = 0 \end{bmatrix}$ $\qquad (p \Rightarrow q)$

$\approx$ ekvivalent

$\begin{bmatrix} f(0) \neq 0 \end{bmatrix} \Rightarrow \begin{bmatrix} f: V \to V \\ \text{är ej linjär} \end{bmatrix}$ $\qquad (\sim q) \Rightarrow (\sim p)$