Ö_6_Mars_1e

den 24 februari 2021 17:48

Kapitel 7 & Kapitel 8

Basuppg. Kap 7: 1, 2, 4, 5, 6
Kap 8: 1, 2, 3

Sidan 227

7.1 $\left\{ \underbrace{\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}}_{v_1}, \underbrace{\begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix}}_{v_2}, \underbrace{\begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}}_{v_3} \right\}$ linjär oberoende?

$\{v_1, v_2, v_3\}$ L. Oberoende $\Leftrightarrow$ $\alpha v_1 + \beta v_2 + \gamma v_3 = 0$ har bara $\alpha = 0, \beta = 0, \gamma = 0$ som lösning.

$$\begin{array}{l} \text{Pivot} \rightarrow \\ L_2 \leftarrow L_2 - 2L_1 \\ R_3 \leftarrow R_3 - 2R_1 \end{array} \begin{bmatrix} 1 & 2 & -1 \\ 2 & 2 & -1 \\ 2 & 5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Gauss elimination och hitta lösning för Homogena system.

$$\text{Pivot} \quad \left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -1 & +1/2 & 0 \\ 0 & 1 & 4 & 0 \end{array} \right) \quad R_2 \leftarrow \frac{1}{2} R_2$$

$$(-1) \left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & +1 & -1/2 & 0 \\ 0 & 0 & 4+1/2 & 0 \end{array} \right) \Rightarrow \left\{ \begin{array}{l} 1x + 2y - 1z = 0 \\ \quad 1y - 1/2 z = 0 \\ \quad \boxed{9/2 z = 0} \Rightarrow \boxed{z = 0} \end{array} \right.$$

Unik lösning

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$y = \frac{1}{2} z \Rightarrow \boxed{y = 0}$

$1x = -2y + z = -2(0) + 0 =$

$\boxed{x = 0}$

Då $\{v_1, v_2, v_3\}$ är linjär oberoende.

Enligt satsen 7.14 n oberoende vektor på $V = \mathbb{R}^n$ dim n
Då $\{v_1 \ldots v_n\}$ spänn upp rummet.

Då är $\{v_1, \ldots \underline{\underline{v_3}}\}$ bas för $\mathbb{R}^3$

2) $\left\{ \begin{pmatrix}1\\2\\2\end{pmatrix}, \begin{pmatrix}2\\2\\5\end{pmatrix}, \begin{pmatrix}1\\4\\1\end{pmatrix} \right\}$

$$\left[\begin{array}{ccc|c}1&2&1&0\\2&2&4&0\\2&5&1&0\end{array}\right] \quad \begin{array}{l}\text{Pivot}\\ R_2 \leftarrow R_2 - 2R_1\\ R_3 \leftarrow R_3 - 2R_1\end{array}$$

$$(-\tfrac{1}{2}) \rightarrow \left[\begin{array}{ccc|c}1&2&1&0\\0&-2&2&0\\0&1&-1&0\end{array}\right] \rightarrow \left[\begin{array}{ccc|c}1&2&1&0\\0&1&-1&0\\0&1&-1&0\end{array}\right] \quad R_3 \leftarrow R_3 - R_2$$

$$\left[\begin{array}{ccc|c}1&2&1&0\\0&1&-1&0\\0&0&0&0\end{array}\right]$$

fri kolumn $\Rightarrow$ fri parameter $(\underline{x_3 = t})$

v_3 är linjär kombination av v_1 och v_2

Då $\{v_1, v_2, v_3\}$ är linjär Beroende

$\therefore$ ej Bas.

7.4 $v_1 = \begin{pmatrix}1\\2\\2\end{pmatrix}$ $v_2 = \begin{pmatrix}1\\4\\1\end{pmatrix}$ $v_1 \cdot v_2 \neq 0$

$v_3 \neq \alpha v_1 + \beta v_2 \Rightarrow \begin{array}{l}\alpha = 1\\ \beta = 1\end{array} \quad \begin{pmatrix}1\\2\\2\end{pmatrix} + \begin{pmatrix}1\\4\\1\end{pmatrix} \neq \begin{pmatrix}2\\-6\\3\end{pmatrix}$ exempel

$v_1 \times v_2 = v_3 \quad v_3 \perp v_1 \;\&\; v_3 \perp v_2$

Bas: $\{v_1, v_2, v_4\}$ v_4 ej linjär komb av $\{v_1, v_2\}$

$$\left(\begin{array}{cc|c|c}1&2&&0\\4&6&&0\\1&-3&&0\end{array}\right) \quad \begin{array}{l}\text{Pivot Row}\\ R_2 \leftarrow R_2 - 2R_1\\ R_3 \leftarrow R_3 - 2R_1\end{array}$$

$$\left(\begin{array}{cc|cc}1&1&2&a\\0&1&(b-2a)/2&\\0&-1&-7&c-2a\end{array}\right) \quad \begin{array}{l}R_1 \leftarrow R_1 - R_2\\ \tfrac{1}{2} \quad \text{pivot}\\ R_3 \leftarrow R_3 + R_2\end{array}$$

$$\left(\begin{array}{ccc|c}1&0&1&a - \left(\frac{b-2a}{2}\right)\\ 0&1&1&(b-2a)/2\end{array}\right)$$

$-a + \dfrac{b-2a}{2} = \dfrac{b}{2} - a - a$
$= \dfrac{b}{2} - 2a$

$$-a+\frac{b-2a}{2} = \frac{b}{2}-a-a$$
$$= \frac{b}{2}-2a$$

$$\frac{(b-2a)}{2} = \left(\frac{b}{2}-a\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & a-\left(\frac{b-2a}{2}\right) \\ 0 & 1 & 1 & (b-2a)/2 \\ 0 & 0 & \cancel{-6}\ 1 & \frac{(b-a)}{2}+c-2a \end{array}\right) \quad R_3 \leftarrow -\frac{1}{6}R_3$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & \left(\frac{b}{2}-a\right)+\frac{1}{6}c+\frac{b}{12}-\frac{1}{2}a \\ 0 & 1 & 0 & \left(\frac{b}{2}-a\right) \\ 0 & 0 & 1 & -\frac{1}{6}c-\frac{b}{12}+\frac{1}{2}a \end{array}\right) \quad \begin{array}{l} R_1 \leftarrow R_1-R_3 \\ R_2 \leftarrow R_2-R_3 \\ \text{Pivot} \end{array}$$

$$\frac{3}{2}a-\frac{5}{12}b+\frac{1}{6}c$$
$$-\frac{3}{2}a+\frac{7}{12}b+\frac{1}{6}c \quad =$$
$$\frac{1}{2}a \quad -\frac{1}{12}b \quad -\frac{1}{6}c$$

$$\underbrace{\begin{pmatrix} 3/2 & -5/12 & 1/6 \\ -3/2 & 7/12 & 1/6 \\ 1/2 & -1/12 & -1/6 \end{pmatrix}}_{A^{-1} = \text{Inverse of } A}\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$Ax = b$$

A inversible

$$\boxed{x = A^{-1}b}$$

Ex 5. page 228.

$E = (e_1, e_2)$ standardbas

$F = (f_1, f_2)$

$G = (g_1, g_2)$

$$f_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad f_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 3 \end{pmatrix} \neq \alpha \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$g_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad g_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

1) $F = \{f_1, f_2\}$ $\begin{cases} \text{linjär oberoende} \\ \text{spänns upp } \mathbb{R}^2 \end{cases}$ $\Rightarrow$ F is a bas för $\mathbb{R}^2$

$G = \{g_1, g_2\}$ $\begin{cases} \text{linjär oberoende} \\ \text{spänns upp } \mathbb{R}^2 \end{cases}$

1-a) $$\left(\begin{array}{cc|c|c} \overbrace{1 \quad 3}^{F} & & 0 & a \\ 3 \quad 1 & & 0 & b \end{array}\right) \xrightarrow[\text{GElim}]{} \left(\begin{array}{cc|c|c} 1 & 3 & 0 & a \\ 0 & 1-3(3) & 0-0 & b-3(a) \end{array}\right)$$

1-a) $\left(\begin{array}{cc|c|c} 1 & 3 & 0 & a \\ 3 & 1 & 0 & b \end{array}\right) \xrightarrow[R_2 \leftarrow R_2 - 3R_1]{\text{GElim}} \left(\begin{array}{cc|c|c} 1 & 3 & 0 & a \\ 0 & \underbrace{1-3(3)}_{\underbrace{1-9}_{-8}} & 0-0 & b-3(a) \end{array}\right)$

$$\left(\begin{array}{cc|c|c} 1 & 3 & & a \\ 0 & \frac{-8}{-8} & 0 & \frac{b-3a}{-8} \end{array}\right) \to \left(\begin{array}{cc|c|c} 1 & 3 & 0 & a \\ 0 & 1 & 0 & \frac{3a-b}{8} \end{array}\right)_{\text{pivot}}$$

$$\left(\begin{array}{cc|c|c} 1 & 3-3 & 0 & a - 3\left(\frac{3a-b}{8}\right) \\ 0 & 1 & 0 & \frac{3a-b}{8} \end{array}\right) \leftarrow \frac{8a}{8} - \frac{9a}{8} + \frac{3b}{8}$$

$$\left(\begin{array}{cc|c|c} 1 & 0 & 0 & -\frac{1}{8}a + \frac{3}{8}b \\ 0 & 1 & 0 & \frac{3}{8}a - \frac{1}{8}b \end{array}\right)$$

$$\begin{pmatrix} -1/8 & 3/8 \\ 3/8 & -1/8 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}$$

$$\underbrace{-\frac{1}{8}\begin{pmatrix} 1 & -3 \\ -3 & 1 \end{pmatrix}}_{(F^{-1})}\begin{pmatrix} a \\ b \end{pmatrix}$$

7.5

b) $x_E = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \qquad x_F = ? \qquad x_G = ?$

$$\boxed{F \cdot x_F = x_E} \Rightarrow \boxed{x_F = F^{-1} \cdot x_E}$$

$$-\frac{1}{8}\begin{pmatrix} 1 & -3 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \underbrace{\begin{pmatrix} 7/8 \\ 3/8 \end{pmatrix}}_{x_F}$$

$$\boxed{x_F = \frac{7}{8} f_1 + \frac{3}{8} f_2}$$

$$\boxed{G \overset{?}{x_G} = x_E}$$

$$\boxed{x_G = \underline{G^{-1}}\, x_E} = -\frac{1}{3}\begin{pmatrix} 1 & -1 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$x_G = \begin{pmatrix} 1/3 \\ 7/3 \end{pmatrix}_G = \underbrace{\tfrac{1}{3}}\, g_1 + \underbrace{\tfrac{7}{3}}\, g_2$$

Kontrol:

$$x_F = \frac{7}{8}\begin{pmatrix} 1 \\ 3 \end{pmatrix} + \frac{3}{8}\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} (9+7)/8 \\ (21+3)/8 \end{pmatrix} = \begin{pmatrix} 16/8 \\ 24/8 \end{pmatrix} = \underbrace{\begin{pmatrix} 2 \\ 3 \end{pmatrix}}_{x_E}$$

$$x_G = \frac{1}{3} g_1 + \frac{7}{3} g_2 = \frac{1}{3}\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \frac{7}{3}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1/3 + 7/3 \\ 2/3 + 7/3 \end{pmatrix} = \begin{pmatrix} 6/3 \\ 9/3 \end{pmatrix} = \underbrace{\begin{pmatrix} 2 \\ 3 \end{pmatrix}}_{x_E}$$

c) $x_E = ?$ $x_G = ?$ given $x_F = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$$(*) \quad \underbrace{G}_{(g_1\ g_2)} x_G = x_E = \underbrace{F x_F}_{(f_1\ f_2)\, x_F}$$

$$\boxed{x_G = G^{-1} F x_F} \qquad x_G = -\frac{1}{3}\begin{pmatrix} 1 & -1 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} -2/3 \\ 31/3 \end{pmatrix} = -\frac{1}{3}\begin{pmatrix} 2 \\ -31 \end{pmatrix}$$

$$\boxed{x_E = F x_F} \qquad \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 11 \\ 9 \end{pmatrix}$$

d) $x_E = ?$ $x_F = ?$ given $x_G = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$$F^{-1} G x_G = x_E = \cancel{F^{-1}} F x_F \quad \leftarrow$$

$$\boxed{x_F = F^{-1} G x_G}$$

$$x_F = \underbrace{-\frac{1}{8}\begin{pmatrix} 1 & -3 \\ -3 & 1 \end{pmatrix}}_{F^{-1}}\begin{pmatrix} -1 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 5/2 \\ -1/2 \end{pmatrix}$$

$$\rightarrow \quad \underbrace{G}\, x_G = x_E$$

$$\begin{pmatrix} -1 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -2+3 \\ 4+3 \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 1 \\ 2 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} 2 \\ 3 \end{pmatrix}}_{X_G} = \begin{pmatrix} -2+3 \\ 4+3 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 \\ 7 \end{pmatrix}}_{X_E}$$

7.6 $\{v_1 \; v_2 \; v_3\}$ ON-bas

you have to show $\|v_1\| = 1$, $\|v_2\| = 1$ $\|v_3\| = 1$

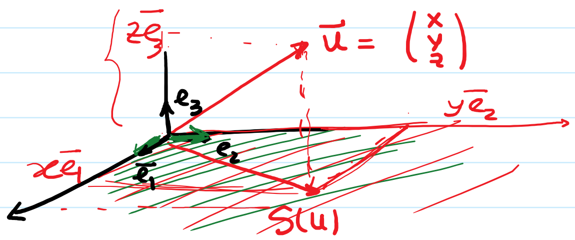
$v_1 \cdot v_2 = 0$ $v_1 \cdot v_3 = 0$ $v_2 \cdot v_3 = 0$

Alex request:

8.2 S = linjär avbildning i $\mathbb{R}^3$

ortogonal projektion på planet $z=0$

Lösning



$\boxed{\Pi_{z=0}} = \left\{ (a, b, 0) \,/\, \begin{matrix} a \in \mathbb{R} \\ b \in \mathbb{R} \end{matrix} \right\}$

$$u = (x, y, z) = x\vec{e}_1 + y\vec{e}_2 + z\vec{e}_3$$

$$S(u) = x\, S\bar{e}_1 + y\, S\bar{e}_2 + z\, S(\vec{e}_3)$$

$$\begin{pmatrix} | & | & | \\ Se_1 & Se_2 & Se_3 \\ | & | & | \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$$

$Se_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 1\,\bar{e}_1$ $\boxed{S\vec{e}_1 = 1\,\vec{e}_1}$ $\lambda_1 = 1$ egenvärde, $e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$Se_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 1\,\vec{e}_2$ $\boxed{S\bar{e}_2 = 1\,\vec{e}_2}$ $\lambda_2 = 1$, $e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$Se_3 = 0 e_3$ $\quad$ $\lambda = 0$ is eigenvalue

e_3 is eingenvector

$$\text{Spectrum} = \{\underbrace{\lambda_1, \lambda_2}, \lambda_3\} = \{1, 0\}$$

$$P = \{\bar{e}_1, \bar{e}_2, \bar{e}_3\}$$ ON-base av egenvektorer

8.1 $\quad A_{2\times 2}$ $\quad \lambda_1 = 2 \quad v_1 = \begin{pmatrix}1\\3\end{pmatrix}$

$\lambda_2 = -1 \quad v_2 = \begin{pmatrix}4\\1\end{pmatrix}$

a) $v_1 \quad Av_1 = \lambda_1 v_1 \Rightarrow (cv_1)$ is also egenvektor till A

$$\|v_1\| = \sqrt{1+9} = \sqrt{10}$$

$$\frac{v_1}{\|v_1\|} = \frac{1}{\sqrt{10}}\begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}1/\sqrt{10}\\3/\sqrt{10}\end{pmatrix}$$ enhetsvektor som är egenvektor till A motsvarande $\lambda_1 = 2$.

$$\frac{v_2}{\|v_2\|} = \frac{1}{\sqrt{16+1}}\begin{pmatrix}4\\1\end{pmatrix} = \begin{pmatrix}4/\sqrt{17}\\1/\sqrt{17}\end{pmatrix}$$

b) A^2

$$AAv = A(\lambda v) = \lambda(Av) = \lambda . \lambda v = \lambda^2 v$$

v egenvektor till A med $\lambda \in \mathbb{R}$

$$\boxed{A^2 v = \lambda^2 v}$$

$$\boxed{Av = \lambda v}$$

$A^2 \cdot \begin{pmatrix}1\\3\end{pmatrix} = (2)^2 \begin{pmatrix}1\\3\end{pmatrix} \quad \tilde{\lambda}_1 = 4$

$A^2 \begin{pmatrix}4\\1\end{pmatrix} = (-1)^2 \begin{pmatrix}4\\1\end{pmatrix} \quad \tilde{\lambda}_2 = 1$

c) A^5 $\quad , \quad Av = \lambda v$

$$A^5 v = \lambda^5 v \qquad \begin{cases} \lambda_1 = 2 \implies (\lambda_1)^5 = 2^5 = 32 \\ \lambda_2 = -1 \implies (\lambda_2)^5 = (-1)^5 = \underline{-1} \end{cases}$$

$$A^5 \begin{pmatrix} 1 \\ 3 \end{pmatrix} = 32 \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$A^5 \begin{pmatrix} 4 \\ 1 \end{pmatrix} = -1 \cdot \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

d) A^{-1} $\qquad \boxed{Av = \lambda v}$

$$A^{-1} \cdot A = I$$

$$v = Iv = A^{-1}(Av) = A^{-1}(\lambda v) = \lambda A^{-1} v \implies$$

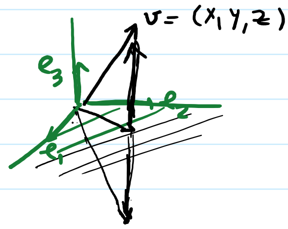
$$\boxed{A^{-1} v = \frac{1}{\lambda} v}$$

$$A^{-1} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 \\ 3 \end{pmatrix} \qquad (\lambda = 2) \quad \frac{1}{\lambda} = \frac{1}{2}$$

$$A^{-1} \begin{pmatrix} 4 \\ 1 \end{pmatrix} = -\frac{1}{1} \begin{pmatrix} 4 \\ 1 \end{pmatrix} \qquad \frac{1}{\lambda} = \frac{1}{-1}$$

8.3 $\quad S =$ spegling i planet $\Pi_{z=0}$

$$E = \{\underbrace{e_1\ e_2}\ e_3\}$$



$\Pi_{z=0}$

$$S(\bar{e}_1) = 1\,\bar{e}_1$$

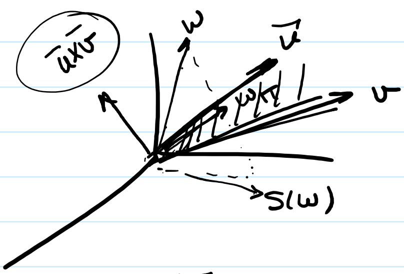
$$S(\bar{e}_2) = 1\,\vec{e}_2$$

$$S(\bar{e}_3) = -1\,\vec{e}_3$$

$$\underbrace{S(v) = (x, y, -z)}$$

$$\text{Spektrum} = \{\underbrace{\lambda_1, \lambda_2}, \lambda_3\} = \{1, -1\}$$

$$\text{EgenVektors} = \{v_1, v_2, v_3\} = \{\bar{e}_1, \bar{e}_2, \bar{e}_3\}$$



$$S(u) = 1u$$
$$S(v) = 1v$$
$$S(\bar{u} \times v) = -(\bar{u} \times v)$$

$G = \{u, v, \vec{u} \times \bar{v}\}$

$$\bar{w} = xu + yv + z(u \times v)$$

$$S(w) = x\,S(u) + y\,S(v) + z\,S(\bar{u} \times v)$$

$$S(w) = \underbrace{\begin{pmatrix} | & | & | \\ S(u)_G & S(v)_G & S(u \times w)_G \\ | & | & | \end{pmatrix}}_{A_G} \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{w_G}$$

$$A_E = \underbrace{G\,A_G}_{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}} G^{-1}$$

$$S(u)_G = 1u = 1u + 0v + 0(u \times v)$$
$$S(u)_G = (1, 0, 0)_G$$

8.3. Alice

$S: \mathbb{R}^3 \to \mathbb{R}^3$

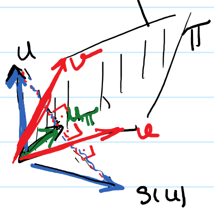
$u \mapsto S(u) =$ spegling(u) i planet $\Pi_{x+y-z=0}$

- Bestäm S_G, med G = bas som består av egenvektors av S.
- Bestäm S_E, $E \equiv$ standard bas.

Den uppgift är samma som 8.3, men planet är inte längre den standard plan $z = 0$.

$\vec{N} = (1, 1, -1)$

Nu $\Pi: x + y - z = 0$



1) Hitta $\bar{\eta}$ = normal vektor och $\{u, v\}$ två vektor som skappa Π.

Vi vill en ON-Bas: att simplifiera räkningar.

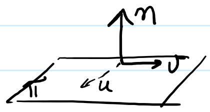
$\Pi: \; x+y-z=0 \Rightarrow x=-y+z \Rightarrow \begin{pmatrix} x\\ y\\ z\end{pmatrix} = \begin{pmatrix} -y+z\\ y\\ z\end{pmatrix} = \begin{pmatrix} -y\\ y\\ 0\end{pmatrix} + \begin{pmatrix} z\\ 0\\ z\end{pmatrix}$

$\begin{pmatrix} x\\ y\\ z\end{pmatrix} = +\overset{t}{\cancel{y}}\begin{pmatrix} -1\\ 1\\ 0\end{pmatrix} + \overset{k}{\cancel{z}}\begin{pmatrix} 1\\ 0\\ 1\end{pmatrix} \quad : \quad u = \begin{pmatrix} -1\\ 1\\ 0\end{pmatrix}, \; v = \begin{pmatrix} 1\\ 0\\ 1\end{pmatrix} \in \Pi$

men $u \cdot v = (-1) \neq 0$ $\quad u, v$ inte ortogonala.

- Så har vi $v \perp \eta$. Nu behöver vi hitta en andra vektor u som är ortogonal mot v och tillhör till planet Π.

$u = v \times \eta$



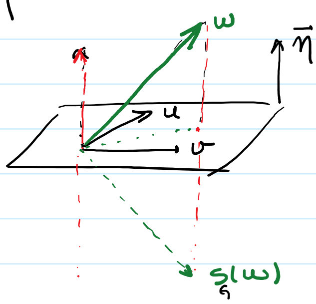
$u = \begin{vmatrix} i & j & k\\ 1 & 0 & 1\\ 1 & 1 & -1\end{vmatrix} = i(-1) - j(-2) + k(1)$

$u = (-1, 2, 1)$

Nu har vi $B = \{v, u, \eta\}$ ortogonal med
$v, u \in \Pi \quad \text{spann}\{v, u\} = \Pi$
$\eta \perp \Pi \quad u \perp v \perp \eta$

Då $G = \left\{ \frac{v}{\|v\|}, \frac{u}{\|u\|}, \frac{\eta}{\|\eta\|} \right\}$ är ON-bas som vi vill.

Därför att Spegling S_G



$w = \alpha u + \beta v + \gamma \eta$

$$\begin{cases} S_G(u) = u \\ S_G(v) = v \\ S_G(\eta) = -\eta \end{cases}$$

$S_G(w) = \alpha S_G(u) + \beta S_G(v) + \gamma S_G(\eta)$

$$\begin{cases} u, v \text{ egenvektorer till } S_G \text{ med } \lambda_1 = \lambda_2 = 1 \\ \eta \text{ egenvektor till } S_G \text{ med } \lambda_3 = -1 \end{cases}$$

$v = (1, 0, 1) \quad \|v\| = \sqrt{2} \Rightarrow \frac{v}{\|v\|} = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

$u = (-1, 2, 1) \quad \|u\| = \sqrt{6} \Rightarrow \frac{u}{\|u\|} = \left(\frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

$\eta = (1, 1, -1) \quad \|\eta\| = \sqrt{3} \Rightarrow \frac{\eta}{\|\eta\|} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$

$$G = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{6} & 1/\sqrt{3} \\ 0 & 2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & -1/\sqrt{3} \end{bmatrix}$$

Då kan vi räkna S_E = spegling relativt E = standard bas

Då kan vi räkna S_E = spegling relativt E = standard bas med hjälp av den Bas byte sats

$$S_E = G\, S_G\, G^{-1} = G\, S_G\, G^t$$ eftersom $G^{-1} = G^t$ (G = ON-bas)

$$S_E = G \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -1/\sqrt{6} & 2/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & -1/\sqrt{3} \end{pmatrix} =$$

$$S_E = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{6} & 1/\sqrt{3} \\ 0 & 2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & -1/\sqrt{3} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -1/\sqrt{6} & 2/\sqrt{6} & 1/\sqrt{6} \\ -1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}$$

$$S_E = \begin{pmatrix} 1/2 + 1/6 - 1/3 & 0 - 2/6 - 1/3 & 1/2 - 1/6 + 1/3 \\ 0 - 2/6 - 1/3 & 0 + 4/6 - 1/3 & 0 + 2/3 + 1/3 \\ 1/2 - 1/6 + 1/3 & 0 + 2/6 + 1/3 & \frac{1}{2} + \frac{1}{6} - 1/3 \end{pmatrix}$$

$$S_E = \begin{bmatrix} 1/3 & -2/3 & 2/3 \\ -2/3 & 1/3 & 2/3 \\ 2/3 & 2/3 & 1/3 \end{bmatrix}$$

$$S_E = \frac{1}{3} \begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$ ■

Kontroll : vi konstruerar S_G så att $S_G \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_G = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_G$, $S_G \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_G = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_G$ $S_G \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_G = -\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_G$

Nu måste vi hitta den här beteende också med koordinater i E.

$$S_E \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{bmatrix} 1/3 & -2/3 & 2/3 \\ -2/3 & 1/3 & 2/3 \\ 2/3 & 2/3 & 1/3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/3 + 0 + 2/3 \\ -2/3 + 0 + 2/3 \\ 2/3 + 0 + 1/3 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 1v$$

$$S_E \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = \begin{bmatrix} 1/3 & -2/3 & 2/3 \\ -2/3 & 1/3 & 2/3 \\ 2/3 & 2/3 & 1/3 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1/3 - 4/3 + 2/3 \\ 2/3 + 2/3 + 2/3 \\ -2/3 + 4/3 + 1/3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = 1u$$

$$S_E \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{bmatrix} 1/3 & -2/3 & 2/3 \\ -2/3 & 1/3 & 2/3 \\ 2/3 & 2/3 & 1/3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1/3 - 2/3 - 2/3 \\ -2/3 + 1/3 - 2/3 \\ 2/3 + 2/3 - 1/3 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = -h$$