CHALMERS

UNIVERSITY OF TECHNOLOGY

SSY281 - MODEL PREDICTIVE CONTROL

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Lecture 1: Introduction and course overview

Goals for today:

- To know about the background of MPC and the ideas behind the course
- To get motivated by being reminded about the limitations of linear design
- To understand the receding horizon control (RHC) idea
- To understand how the course is organized
- To understand the learning objectives of the course

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  - Improvements in numerical computations.

Example: feedback control of a double integrator system

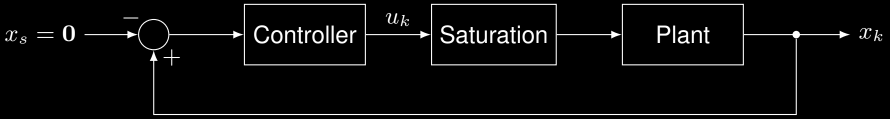
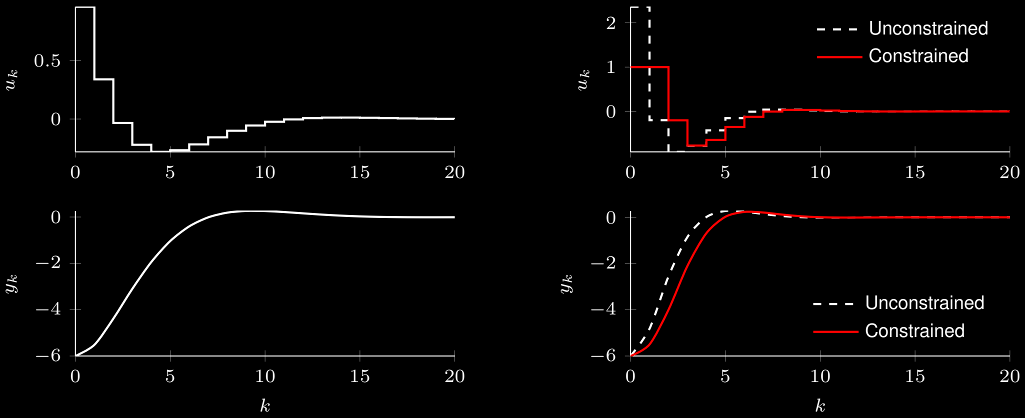


Figure 1: Feedback control loop with input saturation.

The control input is to be kept within bounds $u_k \in [-1, 1], \forall k$. For safety reasons, a saturation function is imposed

$$\mathrm{sat}(u) \triangleq \begin{cases} 1, & u > 1 \\ u, & |u| \leq 1 \\ -1, & u < -1. \end{cases}$$

LQ control of the double integrator system

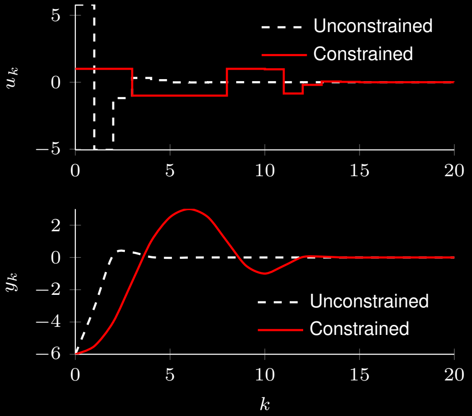


(a) Cautious design with $u_k = -Kx_k$ and $R = 20$.

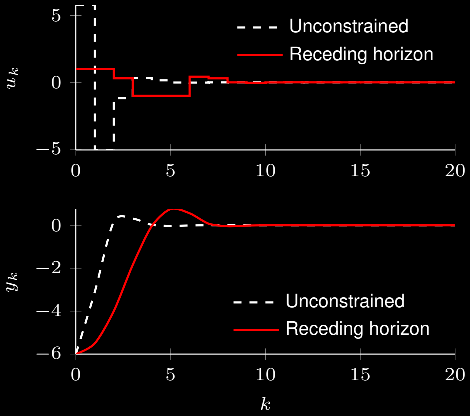
(b) Serendipitous design for constrained and unconstrained LQR with $u_k = -Kx_k$ and $u_k = -\mathrm{sat}(Kx_k)$, respectively, and $R = 2$.

Figure 2: LQ control of the double integrator system.

LQ vs. receding horizon control



(a) Serendipitous design for constrained and unconstrained LQR with $u_k = -Kx_k$ and $u_k = -\mathrm{sat}(Kx_k)$, respectively, and $R = 0.1$.



(b) Unconstrained LQR with $u_k = -Kx_k$ and receding horizon control with $R = 0.1$.

Figure 3: Comparison between LQ and receding horizon control of the double integrator system.

The receding horizon idea

a) At time instant k, *predict* the process response over a finite **prediction horizon** N; this response depends on the sequence of future control inputs over the **control horizon** M.

b) Pick the control sequence which gives the best performance in terms of a specified **objective**, **cost function** or **criterion**.

c) Apply the first element in the control sequence to the process, discard the rest of the sequence, and return to step 1.

An illustration of a receding horizon control

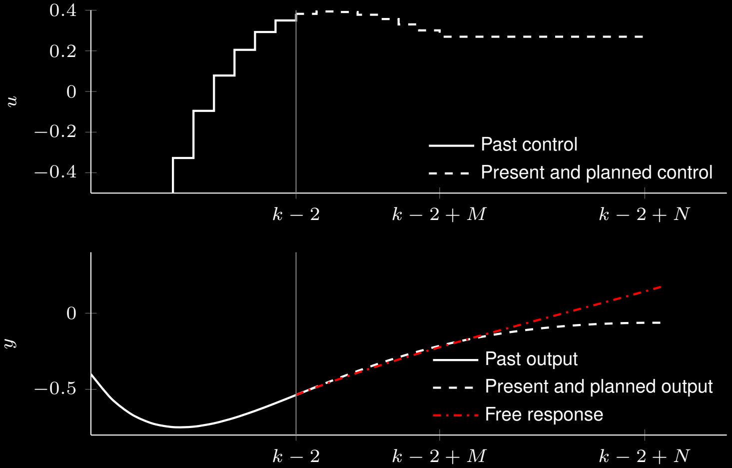


Figure 4: An illustration of a receding horizon control. The free response is the future response when the control signal stays at its current level.

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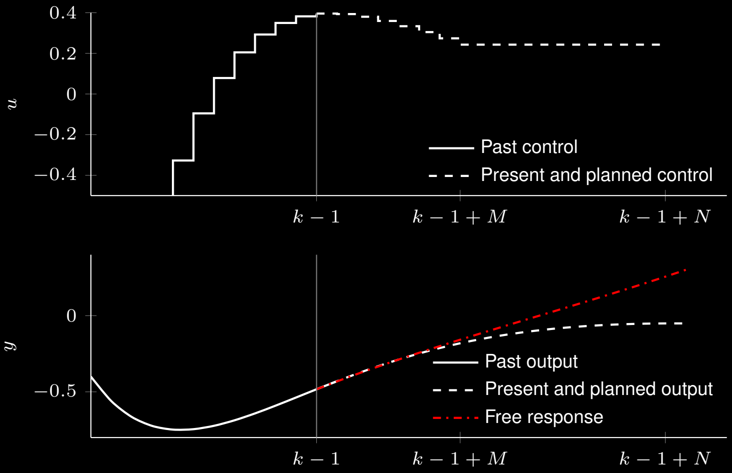

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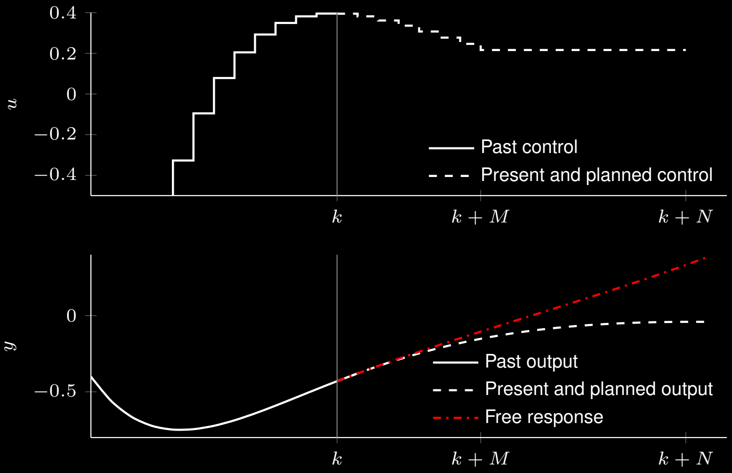

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Example: Receding horizon control of an integrator system

- **System** at time k, with output dynamics: $y(k+1) = y(k) + u(k)$.
- **Past inputs**: $\{\dots, u(k-2), u(k-1)\}$.
- $u(k+1|k)$: **planned (future) control input** at time $k+1$, given information at time k.
- $\Delta u(k+1|k) = u(k+1|k) - u(k|k)$: **control increment**.
- $\hat{y}(k+1|k)$: **predicted output** at time $k+1$ given information at time k.
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The predicted outputs can be written as:

$$\hat{y}(k+1|k) = y(k) + u(k|k) = y(k) + u(k-1) + \Delta u(k|k) = y_f(k+1|k) + \Delta u(k|k)$$

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where $y_f(\cdot|k)$ is the **free response**, i.e. the predicted output if the control input is frozen at time k to its last value $u(k-1)$.

Case 1: control horizon of $M = 1$

Only one future control input is to be chosen, and we assume that the control stays constant after that, i.e. $\Delta u(k+1|k) = 0$. A natural criterion for having future outputs close to the reference signal

$$\begin{aligned} V_2 &= (\hat{y}(k+1|k) - r(k+1))^2 + (\hat{y}(k+2|k) - r(k+2))^2 \\ &= (y_f(k+1|k) + \Delta u(k|k) - r(k+1))^2 + (y_f(k+2|k) + 2\Delta u(k|k) - r(k+2))^2. \end{aligned}$$

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We can solve for the optimal value by differentiating V_2, and set the derivative equal to zero:

$$\frac{\partial V_2}{\partial \Delta u(k|k)} = 2(y_f(k+1|k) + \Delta u(k|k) - r(k+1)) + 2(y_f(k+2|k) + 2\Delta u(k|k) - r(k+2)) \cdot 2 = 0.$$

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This gives the optimal (incremental) control (remember that $u(k) = u(k-1) + \Delta u(k|k)$)

$$\Delta u(k|k) = \frac{1}{5}\left((r(k+1) - y_f(k+1|k)) + 2(r(k+2) - y_f(k+2|k))\right). \tag{1}$$

Alternatively, we could describe this procedure as trying to solve the system of linear equations

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or, using vector notation,

$$\begin{bmatrix} y_f(k+1|k) \\ y_f(k+2|k) \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} \Delta u(k|k) = \begin{bmatrix} r(k+1) \\ r(k+2) \end{bmatrix} \Leftrightarrow$$

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Since there is only one variable to fulfil two conditions, a natural solution is to find the value of $\Delta u(k|k)$ that solves this system of linear equations in a least-squares sense, i.e. by minimizing $\|\Theta \Delta u(k|k) - (\boldsymbol{r} - \boldsymbol{y_f})\|^2$. Expressing this in Matlab notation gives

$$\Delta u(k|k) = \Theta \backslash (\boldsymbol{r} - \boldsymbol{y_f}) = (\Theta^\top \Theta)^{-1} \Theta^\top (\boldsymbol{r} - \boldsymbol{y_f}), \quad (2)$$

which is identical to the solution (1).

Case 2: control horizon of $M = 2$

There are two free variables. Hence, with the system of linear equations interpretation, the conditions to be fulfilled now become

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which can now be solved uniquely for the optimal control sequence:

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Sticking to the receding horizon principle, only the first element in the optimal control sequence is used, namely $\Delta u(k|k)$, and the other element is discarded.

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c) Minimize a criterion (now adopting the index notation as in Matlab)

$$V(k) = V(\hat{y}(k+1:k+N|k), u(k:k+M-1|k))$$

with respect to the control sequence $u(k:k+M-1|k)$.

Summary

The MPC recipe for the example:

a) At time k, predict the output N samples ahead:

$$\hat{y}(k+1|k), \dots, \hat{y}(k+N|k).$$

b) The predictions depend on future control inputs

$$u(k|k), u(k+1|k), \dots, u(k+M-1|k).$$

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c) Minimize a criterion (now adopting the index notation as in Matlab)

$$V(k) = V(\hat{y}(k+1:k+N|k), u(k:k+M-1|k))$$

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d) Apply the first control signal in the sequence to the process:

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e) Increment time $k := k+1$ and go to 1.

MPC ingredients

- An **internal model** describing process and disturbances.
- An **estimator/predictor** to determine the evolution of the state.
- An **objective/criterion** to express the desired system behaviour.
- An **online optimisation algorithm** to determine future control actions.
- The **receding horizon** principle.
- Our focus: linear models, quadratic criteria with linear constraints.

Outline of the course

- What is MPC? Introduction, motivation and review.
- MPC basics and relations to dynamic programming.
- How to cope with incomplete state information.
- Solving predictive control problems.
- User aspects, case study, guest lecture.
- MPC theory.
- Review and outlook.

Learning objectives

After completion of the course, you should be able to:

- Understand and explain the basic principles of model predictive control, its pros and cons, and the challenges met in implementation and applications.
- Correctly state, in mathematical form, MPC formulations of control problems in various applications.
- Describe and construct MPC controllers based on a linear model, quadratic costs and linear constraints.
- Describe basic properties of MPC controllers and analyse algorithmic details on simple examples.
- Understand and explain basic properties of the optimisation problem as an ingredient of MPC, in particular concepts like linear, quadratic and convex optimisation, optimality conditions, and feasibility.
- Use software tools for analysis and synthesis of MPC controllers.

Practical information

- Canvas: syllabus, news, schedule, assignments and more...
- Literature.
- Lectures: slides and Lecture Notes. Lectures will be recorded.
- Problem solving sessions: analytical problems + Matlab coding.
- Home assignments.
- Micro assignments.
- Examination.
- Course representatives.

Literature

- *Model Predictive Control, Lecture Notes*.
- J.B. Rawlings, D.Q. Mayne, M.M. Diehl: *Model Predictive Control. Theory, Computation, and Design*, 2nd ed. Amazon or online.
- Supplementary literature suggested at the course homepage.
- Also @Canvas: Assignments, Matlab files, etc.

Home and micro assignments

There are **7 mandatory home assignments** and 1 optional. The home assignments constitute the main part of the examination in the course and should be pursued **individually**.
Note:

- Read the instructions carefully.
- Start working early to meet deadlines.
- Use Supervision sessions to get help.

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Note:

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- Use Supervision sessions to get help.

There are also **13 compulsory micro-assignments** to be completed before each lecture.

- They are designed in the form of quizzes, to be completed online, on Canvas.
- Each micro-assignment gives maximum of 2 points.
- At least 18 points are needed in order to pass the course.

Grading scheme for home assignments

- Each assignment gives maximum 15 points, except assignment 1 that gives max 10 points.
- The seven compulsory assignments thus give in total max 100 points.
- The optional assignment 8 can give additional 15 points.
- The total score decides the grade with limits 40, 60, 80 points.
- To ensure enough coverage of the course, a minimum of 5 points is required for each of at least 6 assignments.

Each assignment has a deadline. Possible late submissions are handled in the following way:

- A cumulative ”delay budget” of max 5 days is available for each student.
- Total delay within this limit will not affect the score, but once the limit is reached, assignments submitted late will render 0 points, as will any assignment submitted more than 3 days late.

Course representatives

The following student representatives will participate in the course evaluation:

- Alfred Hazard
- Subramanya Mallappa
- Nathaly Sanchez Chan
- Pramod Sivaramakrishnan
- Ahmad Wahba

References

[1] J.B. Rawlings, D.Q. Mayne, and M.M. Diehl. *Model Predictive Control: Theory, Computation, and Design, 2nd edition*. Nob Hill Publishing 2017.
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[2] G. Goodwin, M.M. Seron, and J.A. De Don·. *Constrained Control and Estimation*. Springer 2004.
Available online via Chalmers Library.

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Available online at http://www.mpc.berkeley.edu/mpc-course-material

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