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SSY281 - MODEL PREDICTIVE CONTROL

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Lecture 3: Unconstrained receding horizon control

Goals for today:

- To master the formulation of linear quadratic control (LQ)
- To understand how dynamic programming can be used to solve the LQ problem
- To formulate and solve the finite-time LQ problem using the “batch approach”
- To formulate an unconstrained receding horizon control based on LQ

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Learning objectives:

- Correctly state, in mathematical form, MPC formulations based on descriptions of control problems expressed in application terms
- Describe and construct MPC controllers based on a linear model, quadratic costs and linear constraints
- Describe basic properties of MPC controllers and analyse algorithmic details on very simple examples

The LQ problem

System:

$$S: \quad x^+ = Ax + Bu. \tag{14}$$

Optimisation problem:

$$P: \quad \min_{u(0:N-1)} V_N(x(0), u(0:N-1))$$

where the minimization is with respect to the sequence of control inputs

$$u(0:N-1) = \{u(0), u(1), \ldots, u(N-1)\}$$

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The *objective* or *criterion* or *cost function* V_N is given by

$$\begin{aligned} V_N(x(0), u(0:N-1)) &= x^\top(N) P_f x(N) + \sum_{i=0}^{N-1} (x^\top(i) Q x(i) + u^\top(i) R u(i)) \\ &= l_f(x(N)) + \sum_{i=0}^{N-1} l(x(i), u(i)). \end{aligned} \tag{15}$$

Batch solution

Repeated use of the system equation (14) gives

$$\begin{bmatrix} x(1) \\ x(2) \\ \vdots \\ x(N) \end{bmatrix} = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^N \end{bmatrix} x(0) + \begin{bmatrix} B & 0 & \cdots & 0 \\ AB & B & \ddots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-1}B & A^{N-2}B & \cdots & B \end{bmatrix} \begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(N-1) \end{bmatrix} \tag{16}$$

or, with a more compact notation,

$$\boldsymbol{x} = \Omega x(0) + \Gamma \boldsymbol{u}. \tag{17}$$

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The LQ criterion (15) can now be written

$$V_N(x(0), \boldsymbol{u}) = x^\top(0) Q x(0) + \boldsymbol{x}^\top \bar{Q} \boldsymbol{x} + \boldsymbol{u}^\top \bar{R} \boldsymbol{u} \tag{18}$$

where $\bar{Q} = \mathrm{diag}(Q, \ldots, Q, P_f)$ and $\bar{R} = \mathrm{diag}(R, \ldots, R)$.

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where $\bar{Q} = \mathrm{diag}(Q, \ldots, Q, P_f)$ and $\bar{R} = \mathrm{diag}(R, \ldots, R)$.

Since $V_N(x(0), \boldsymbol{u})$ is quadratic in $\boldsymbol{u}$, we can solve for the optimal control vector (either by differentiation or by completing the squares):

$$\boldsymbol{u}^* = -(\Gamma^\top \bar{Q}\Gamma + \bar{R})^{-1}\Gamma^\top \bar{Q}\Omega\, x(0)$$

and the optimal **cost-to-go**, or **value function**, is

$$V_N^*(x(0), \boldsymbol{u}) = x^\top(0)\left(Q + \Omega^\top \bar{Q}\Omega - \Omega^\top \bar{Q}\Gamma(\Gamma^\top \bar{Q}\Gamma + \bar{R})^{-1}\Gamma^\top \bar{Q}\Omega\right)x(0).$$

Note that $\boldsymbol{u}^*$ is linear in $x(0)$ and V_N^* is quadratic in $x(0)$!

Dynamic programming solution

Let us inspect the cost function V_N in the same way by spelling out the stage costs:

$$V_N = l(x(0),u(0)) + \ldots + \overbrace{l(x(N-2),u(N-2)) + \overbrace{l(x(N-1),u(N-1)) + l_f(x(N))}^{u(N-1)}}^{u(N-2)}. \tag{19}$$

In the expression above, an important property of the problem has been indicated: each control input affects only a corresponding *tail* of the sum; the later the control input is, the shorter the tail. Starting at the very end, only the last two terms of (19) depend on the last control input $u(N-1)$;

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$$\min_{u(0:N-1)} V_N(x(0), u(0:N-1)) =$$
$$\min_{u(0:N-2)} \left\{ V_{N-1}(x(0), u(0:N-2)) + \min_{u(N-1)} \left[l(x(N-1), u(N-1)) + l_f(x(N)) \right] \right\}.$$

This allows us to start unfolding the solution to the LQ problem by applying *backwards dynamic programming*.

The last two terms of (19) are quadratic in $u(N-1)$. We can rewrite this expression into one quadratic form by using the system model (14) and by completing the squares as follows:

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& \qquad = x^\top(Q + A^\top P_f A)x + u^\top(R + B^\top P_f B)u + 2x^\top A^\top P_f Bu
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&\qquad + x^\top(Q + A^\top P_f A - A^\top P_f B(R + B^\top P_f B)^{-1}B^\top P_f A)x
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u^*(N-1) = u^*(N-1; x) = \underbrace{-(R + B^\top P_f B)^{-1}B^\top P_f A}_{K(N-1)}\, x \equiv K(N-1)x
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$$u^*(N-1) = u^*(N-1; x) = \underbrace{-(R + B^\top P_f B)^{-1}B^\top P_f A}_{K(N-1)}\, x \equiv K(N-1)x$$

and the resulting optimal cost-to-go from state x at time $N-1$ to the final time N is

$$V^*_{N-1\to N}(x) = x^\top \underbrace{(Q + A^\top P_f A - A^\top P_f B(R + B^\top P_f B)^{-1}B^\top P_f A)}_{P(N-1)}\, x \equiv x^\top P(N-1)x. \tag{20}$$

Using the result above, namely that the minimum value of the last two terms in (19) is given as an explicit function of $x(N-1)$ in (20), we now proceed to include one more term from (19), namely the one depending on $u(N-2)$:

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\end{aligned}$$

By carrying out calculations, completely analogous to the ones in step 1 (P_f is replaced by $P(N-1)$), we get the optimal control

$$u^*(N-2) = u^*(N-2;x) = -(R + B^\top P(N-1)B)^{-1}B^\top P(N-1)Ax = K(N-2)x,$$

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where x is now short for $x(N-2)$, and the optimal cost-to-go from state x at time $N-2$ to the final time N is

$$V^*_{N-2\to N}(x) = x^\top P(N-2)x, \tag{21}$$

and $P(N-2)$ is given by the **Riccati equation**

$$P(N-2) = Q + A^\top P(N-1)A - A^\top P(N-1)B(R + B^\top P(N-1)B)^{-1}B^\top P(N-1)A.$$

DP solution to the LQ problem

The sequence of optimal control laws (**control policy**) is computed as:

$$u^*(k;x) = K(k)x(k), \quad k = 0, \dots, N-1$$
$$K(k) = -(R + B^\top P(k+1)B)^{-1} B^\top P(k+1)A$$

where the **Riccati equation** is

$$P(k-1) = Q + A^\top P(k)A - A^\top P(k)B(R + B^\top P(k)B)^{-1} B^\top P(k)A, \quad P(N) = P_f \tag{22}$$

or equivalently

$$P(k-1) = Q + A^\top P(k)A + A^\top P(k)BK(k-1), \quad P(N) = P_f. \tag{23}$$

The **optimal cost-to-go** (from time k to time N) is

$$V^*_{k\to N}(x) = x^\top(k)P(k)x(k).$$

Example: batch solution of a simple integrator system

Consider the simple integrator system

$$x^+ = x + u.$$

We will study the finite-time optimal control problem with $x_0 = 1$, $N = 3$ and $Q = R = P_f = 1$. Using the batch approach, the optimal control sequence can be found as

$$\boldsymbol{u}^* = -\begin{bmatrix} 4 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} x_0 = -\begin{bmatrix} 0.615 \\ 0.231 \\ 0.077 \end{bmatrix} x_0$$

$$\boldsymbol{x}^* = \begin{bmatrix} 1 & 0.385 & 0.154 & 0.077 \end{bmatrix}^\top.$$

DP solution to the integrator system

We may also solve the problem with DP. The solution can be obtained as $u^*(k) = K(k)x(k)$, where the optimal control gain can be found via backward recursion,

$$K(0:2) = (-0.615, -0.6, -0.5)\,.$$

The optimal control sequence can now be obtained by simulating the system forward in time,

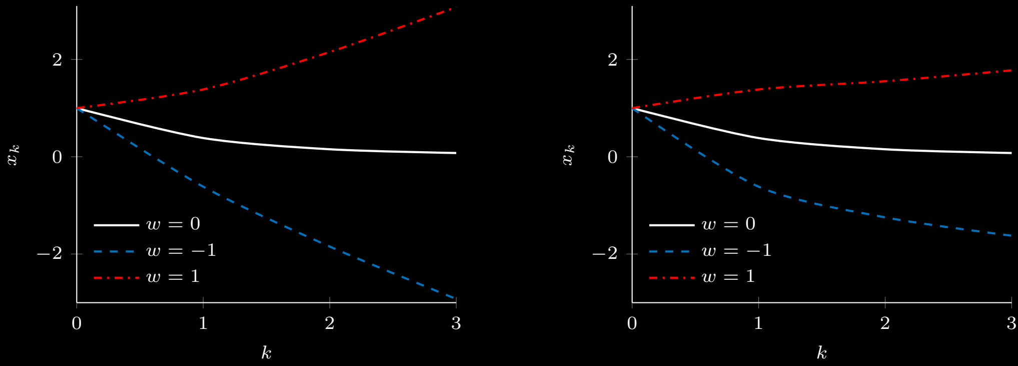
$$\boldsymbol{u}^* = \begin{bmatrix} -0.615 \cdot 1 \\ -0.6 \cdot 0.385 \\ -0.5 \cdot 0.154 \end{bmatrix} = -\begin{bmatrix} 0.615 \\ 0.231 \\ 0.077 \end{bmatrix}$$

$$\boldsymbol{x}^* = \begin{bmatrix} 1 & 0.385 & 0.154 & 0.077 \end{bmatrix}^\top .$$

Clearly, and as expected, the solutions by the batch and DP approach are identical.

Batch vs. DP solution under additive process disturbance

$$x^+ = x + u + w.$$



(a) Open loop control.

(b) Closed-loop, feedback control.

Figure 6: Receding horizon control of an uncertain system with an additive process disturbance w.

Example: optimality does not guarantee stability

Consider the system

$$x^+ = Ax + Bu = \begin{bmatrix} 4/3 & -2/3 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = Cx = \begin{bmatrix} -2/3 & 1 \end{bmatrix} x$$

and design an LQ controller for $Q = P_f = C^\top C + \delta I$ with $\delta = 0.001$ and $R = 0.001$. The time horizon is $N = 5$. To compute the solution, the Riccati equation (22) is iterated 4 times with $P(5) = P_f$, and the controller gain is obtained as

$$K(0) = -(R + B^\top P(1)B)^{-1} B^\top P(1)A = \begin{bmatrix} -0.026 & 0.665 \end{bmatrix}.$$

The eigenvalues of the closed-loop matrix $A + BK(0)$ are $\{1.307, 0.001\}$, i.e. the closed-loop system is unstable.

Infinite horizon LQ control

The system

$$S: \quad x^+ = Ax + Bu \tag{24}$$

with optimisation criterion

$$V_\infty(x(0), u(0\!:\!\infty)) = \sum_{i=0}^{\infty}(x^\top(i)Qx(i) + u^\top(i)Ru(i)) \tag{25}$$

and with pair (A, B) controllable, and $Q, R \succ 0$, is an infinite horizon LQ control problem whose solution gives a stable closed-loop system

$$x^+ = Ax + BKx$$

with a time invariant feedback gain K, given by

$$K = -(B^\top PB + R)^{-1}B^\top PA \tag{26}$$

$$P = Q + A^\top PA - A^\top PB(B^\top PB + R)^{-1}B^\top PA. \tag{27}$$

The latter equation is called the *algebraic Riccati equation*. The optimal cost is given by

$$V_\infty^*(x(0)) = x^\top(0)Px(0).$$

Example: optimality does not guarantee stability, cont'd

Consider again the LQ design for the system in the previous example. If the time horizon is increased and chosen as $N = 7$, i.e. the Riccati equation is iterated two more times, then the eigenvalues of the closed-loop matrix become $\{0.989, 0.001\}$, i.e. the closed-loop system is now just stable. Continuing to iterate the Riccati equation, the solution converges to the solution of the algebraic Riccati equation, giving the infinite horizon closed-loop eigenvalues

$$\mathrm{eig}(A + BK) = \{0.664, 0.001\}$$

i.e. the closed-loop system is stable as predicted.

Example: unstable and stable optimal control

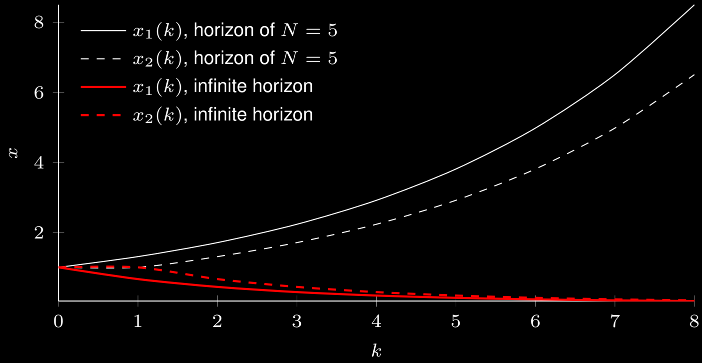


Figure 7: State evolution from the two optimal controllers, one with horizon of $N = 5$ and the other with an infinite horizon.

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