CHALMERS

UNIVERSITY OF TECHNOLOGY

SSY281 - MODEL PREDICTIVE CONTROL

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Lecture 4: Constrained receding horizon control

Goals for today:

- To understand the principles behind and to formulate a constrained receding horizon controller (RHC).
- To formulate an MPC based on linear models and quadratic criteria.

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- To formulate an MPC based on linear models and quadratic criteria.

Learning objectives:

- Describe and construct MPC controllers based on linear model, quadratic costs and linear constraints.
- Describe basic properties of MPC controllers and analyse algorithmic details on simple examples.

Infinite horizon optimal control

With the cost function defined as

$$V_\infty(x_0, u(0{:}\infty)) = \sum_{i=0}^{\infty} l(x(i), u(i)),$$

with $l(\cdot,\cdot) \geq 0$ and $l(0,0) = 0$, the infinite horizon optimal control problem is

$$\min_{u(0:\infty)} \quad V_\infty(x_0, u(0{:}\infty)) \tag{28}$$

$$\text{subject to} \quad x^+ = f(x,u), \quad x(0) = x_0 \tag{29}$$

$$x(k) \in \mathbb{X}, \quad u(k) \in \mathbb{U}, \quad \text{for all } k \in (0,\infty). \tag{30}$$

Finite horizon optimal control

With the cost function defined as

$$V_N(x_0, u(0{:}N-1)) = V_f(x(N)) + \sum_{i=0}^{N-1} l(x(i), u(i)),$$

with the final cost $V_f(\cdot) \geq 0$ and $V_f(0) = 0$, the finite horizon optimal control problem is

$$\min_{u(0:N-1)} \quad V_N(x_0, u(0{:}N-1)) \tag{31}$$

$$\text{subject to} \quad x^+ = f(x, u), \quad x(0) = x_0 \tag{32}$$

$$x(k) \in \mathbb{X}, \quad u(k) \in \mathbb{U}, \quad \text{for all } k \in (0, N-1) \tag{33}$$

$$x(N) \in \mathbb{X}_f \subseteq \mathbb{X}. \tag{34}$$

Note that we have added a *terminal constraint* $x(N) \in \mathbb{X}_f$!
It is assumed that $\mathbb{U}$, $\mathbb{X}$, and $\mathbb{X}_f$ all contain the origin.

Solution to the finite time optimisation problem

Optimal cost-to-go:

$$V_N^*(x_0) = \min_{u(0:N-1)} \{V_N(x_0, u(0:N-1)) \mid u(0:N-1) \in \mathcal{U}_N(x_0)\}.$$

Optimal control and state sequences:

$$u^*(0:N-1; x_0) = \{u^*(0; x_0), u^*(1; x_0), \ldots, u^*(N-1; x_0)\}$$
$$x^*(0:N; x_0) = \{x^*(0; x_0), x^*(1; x_0), \ldots, x^*(N; x_0)\}.$$

Receding horizon control

1. At sampling instant k, when we have access to the state $x(k) = x$ (assumed to be within the feasible set), solve the optimisation problem

$$V_N^*(x) = \min_{u(0:N-1)} \{V_N(x, u(0:N-1)) \mid u(0:N-1) \in \mathcal{U}_N(x)\}$$

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$$u(k) = u^*(0; x).$$

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Note that the receding horizon controller described this way *implicitly* defines a feedback control law for all x belonging to the feasible set

$$u(x) = \kappa_N(x) \equiv u^*(0; x), \quad x \in \mathcal{X}_N.$$

Linear quadratic MPC

The system model is linear,

$$x^+ = Ax + Bu,$$

and the criterion is quadratic as described in Section 3.1. Constraints on control signals due to actuator saturation take the form

$$u_{\min} \leq u(k) \leq u_{\max}, \quad \text{for all } k \geq 0 \tag{35}$$

and similar constraints on the states due to e.g. safety or quality concerns may be

$$x_{\min} \leq x(k) \leq x_{\max}, \quad \text{for all } k \geq 0. \tag{36}$$

Sometimes, it is relevant to be able to put constraints also on the rate of change of control inputs,

$$\Delta u_{\min} \leq u(k) - u(k-1) \leq \Delta u_{\max}, \quad \text{for all } k \geq 0. \tag{37}$$

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Such constraints can easily be put in a form that generalizes the inequalities (35) and (36) by introducing a new state vector

$$\xi(k) = \begin{bmatrix} x(k) \\ u(k-1) \end{bmatrix}$$

for which the augmented system model becomes

$$\xi^+ = \mathcal{A}\xi + \mathcal{B}\Delta u$$

with appropriately defined matrices (see Section 2). The constraint (37) can then be stated as

$$\begin{bmatrix} 0 & -I \\ 0 & I \end{bmatrix} \xi(k) + \begin{bmatrix} I \\ -I \end{bmatrix} u(k) \leq \begin{bmatrix} \Delta u_{\max} \\ -\Delta u_{\min} \end{bmatrix}.$$

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The conclusion is that all the linear constraints described can be written in the compact form

$$F\boldsymbol{u} + G\boldsymbol{x} \leq h \tag{38}$$

with $\boldsymbol{u}$, $\boldsymbol{x}$ defined in (20), for some matrices F, G and some vector h, all of appropriate dimensions.

Linear quadratic MPC: summary

1. At sampling instant k, when we have access to the state $x(k) = x$, solve the optimisation problem

$$V_N^*(x) = \min_{u(0:N-1)} \{V_N(x, u(0:N-1)) \mid u(0:N-1) \in \mathcal{U}_N(x)\},$$

$$V_N(x, u(0:N-1)) = x^\top(N)P_f x(N) + \sum_{i=0}^{N-1} \Big(x^\top(i)Qx(i) + u^\top(i)Ru(i)\Big); \; x(0) = x$$

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Linear quadratic MPC with vector notation: condensed form

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Example: LQ MPC for an integrator

Consider a simple integrator system

$$x^+ = x + u$$

for which we want to design an LQ based MPC with $Q = R = P_f = 1$, $N = 2$ and a control constraint

$$u \in \mathbb{U} = [-1, 1].$$

Using the state equation and the notation $x(0) = x$ as above, we get the following expressions for the state variables involved in the optimisation:

$$x(0) = x \qquad x(1) = x + u(0) \qquad x(2) = x + u(0) + u(1).$$

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The cost function to minimize with respect to $\boldsymbol{u} = [u(0)\ u(1)]^\top$, satisfying the control constraint, is

$$\begin{aligned} V_N(x, \boldsymbol{u}) &= x^2 + (x + u(0))^2 + (x + u(0) + u(1))^2 + u(0)^2 + u(1)^2 \\ &= \boldsymbol{u}^\top H \boldsymbol{u} + 2[2x \ \ x]\boldsymbol{u} + 3x^2 \equiv \boldsymbol{u}^\top H \boldsymbol{u} + 2c(x)^\top \boldsymbol{u} + d(x) \end{aligned}$$

with

$$H = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \qquad c(x) = \begin{bmatrix} 2x \\ x \end{bmatrix} \qquad d(x) = 3x^2.$$

Completing the squares, we can finally rewrite the cost function as

$$V_N(x, \boldsymbol{u}) = (\boldsymbol{u} - a(x))^\top H (\boldsymbol{u} - a(x)) + d(x) - a(x)^\top H a(x)$$

where

$$a(x) = -H^{-1}c(x) = Kx, \quad K = \begin{bmatrix} -3/5 \\ -1/5 \end{bmatrix}.$$

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It is clear from the expression for $V_N(x, \boldsymbol{u})$ that $\boldsymbol{u} = a(x)$ is the global minimizer, and as long as this solution respects the constraints, it is also the solution of the constrained optimisation problem.

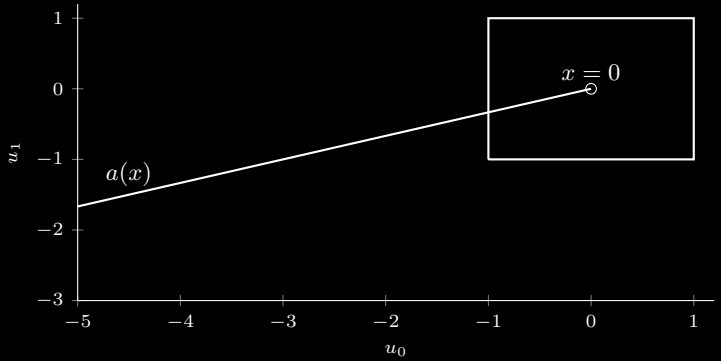
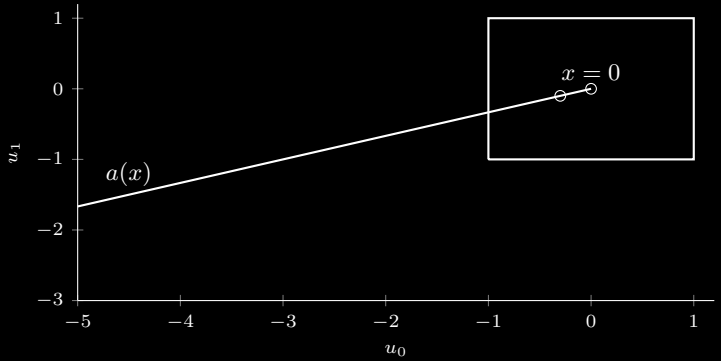


Figure 8: LQ MPC for the integrator system. Here, $a(x)$ is the unconstrained minimizer, and the red lines depict level curves for $V_N(x, \boldsymbol{u})$.



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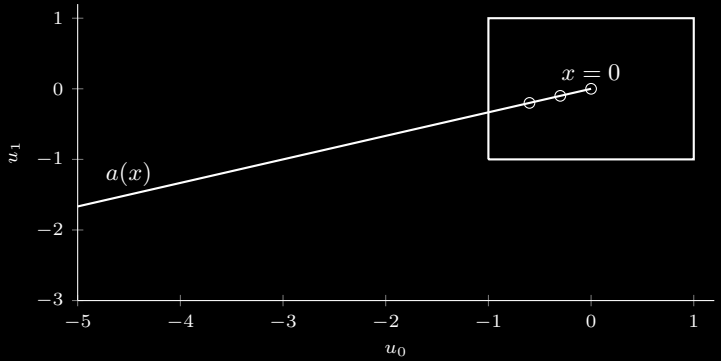

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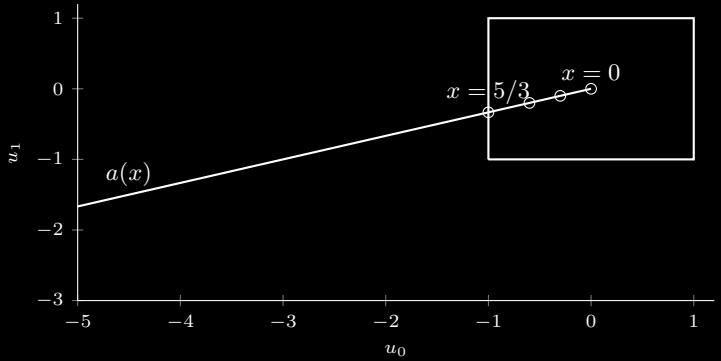

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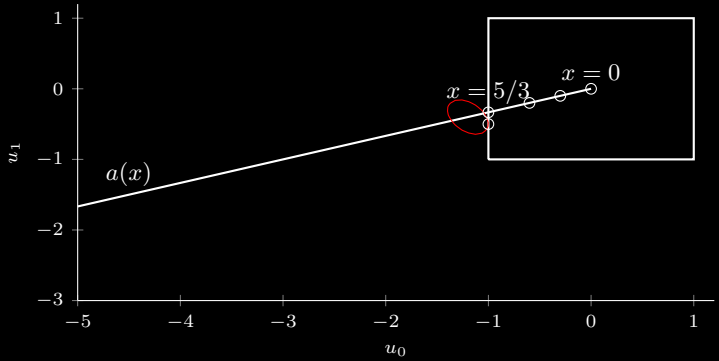

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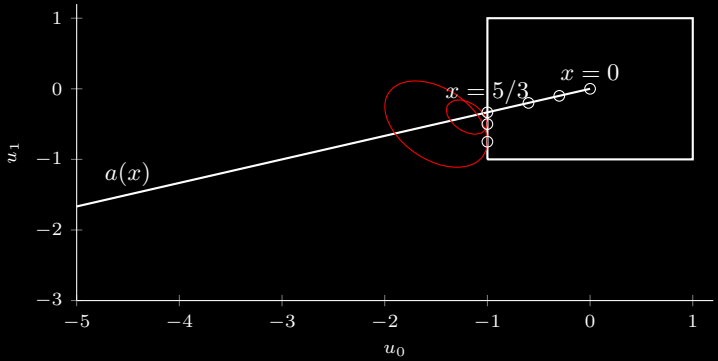

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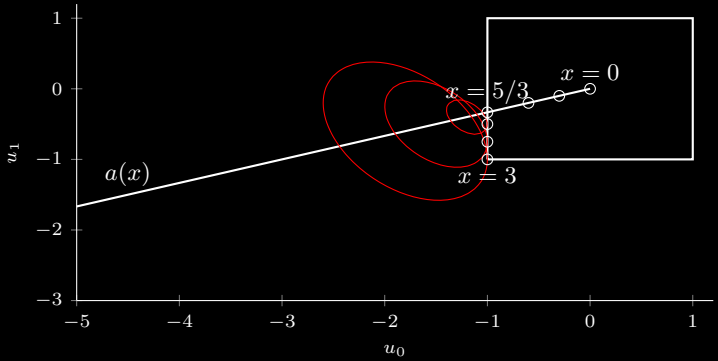

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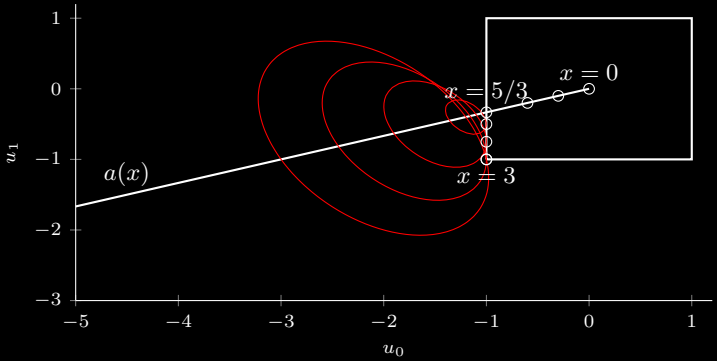

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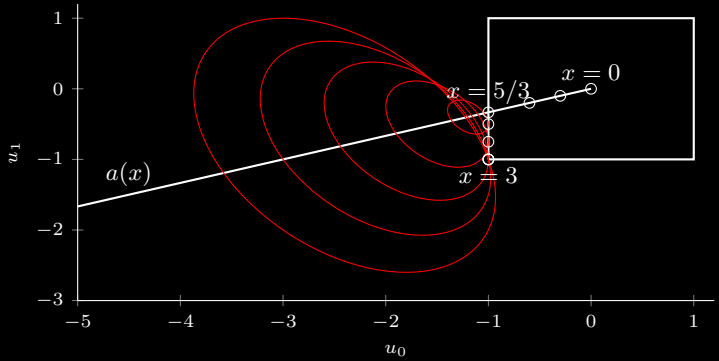

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The conclusion from these arguments is that the receding horizon controller, which is determined from the first component of the minimizing control sequence, i.e. $u(0)$, is given by

$$u(x) = \kappa_N(x) = u^*(0; x) = \begin{cases} -3/5 \cdot x & 0 \leq x \leq 5/3 \\ -1 & x \geq 5/3. \end{cases}$$

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It is then easy to verify that the same arguments hold for negative x, and the resulting control law is indeed symmetric:

$$u(x) = \kappa_N(x) = u^*(0; x) = \begin{cases} 1 & x \leq -5/3 \\ -3/5 \cdot x & -5/3 \leq x \leq 5/3 \\ -1 & x \geq 5/3. \end{cases}$$

Linear quadratic MPC: uncondensed (lifted) formulation

1. At sampling instant k, when we have access to the state $x(k) = x$, solve the optimisation problem

$$V_N^*(x) = \min_{u(0:N-1),x(0:N)} \{V_N(x, u(0:N-1), x(0:N))\},$$

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and subject to

$$x(k+1) = Ax(k) + Bu(k), \quad x(0) = x \tag{39}$$
$$x(k) \in \mathbb{X} \quad u(k) \in \mathbb{U} \quad \text{for all } k \in (0, N-1) \tag{40}$$
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Linear quadratic MPC: uncondensed vector formulation

1. At sampling instant k, when we have access to the state $x(k) = x$, solve the optimisation problem

$$\begin{aligned} \underset{\boldsymbol{u},\boldsymbol{x}}{\text{minimize}} \quad & V_N(x, \boldsymbol{u}, \boldsymbol{x}) = x^\top Q x + \boldsymbol{x}^\top \bar{Q} \boldsymbol{x} + \boldsymbol{u}^\top \bar{R} \boldsymbol{u} \\ \text{subject to} \quad & x^+ = Ax + Bu \\ & F\boldsymbol{u} + G\boldsymbol{x} \leq h. \end{aligned}$$

2. Apply the first control input $u(k) = \boldsymbol{u}(0)$.
3. Let $k := k + 1$ and go to 1.

References

[1] J.B. Rawlings, D.Q. Mayne, and M.M. Diehl. *Model Predictive Control: Theory, Computation, and Design, 2nd edition*. Nob Hill Publishing 2017.
Available online at `https://sites.engineering.ucsb.edu/~jbraw/mpc`

[2] G. Goodwin, M.M. Seron, and J.A. De Don·. *Constrained Control and Estimation*. Springer 2004.
Available online via Chalmers Library.

[3] F Borrelli, A. Bemporad, and M. Morari. *Predictive Control for Linear and Hybrid Systems*.
Available online at http://www.mpc.berkeley.edu/mpc-course-material

[4] J. Maciejowski. *Predictive Control with Constraints*. Prentice Hall 2002.

[5] S. Boyd and L. Vandenberghe. *Convex optimisation*. Cambridge University Press 2004.

[6] Diehl, M. *Real-Time Optimization for Large Scale Nonlinear Processes*. PhD thesis, University of Heidelberg, 2001.

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