SSY281 PSS 5 - MPT Part 1: Reachable and Invariant Sets

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SSY281 PSS 5 - Reachable Sets for Autonomous Systems

Consider the autonomous system:

$$x(k+1) = Ax(k), \tag{1}$$

with constraints $x(k) \in \mathbb{X}, \forall k$.

The (1-)backward reachable set of $\mathbb{X}$ is:

$$\mathrm{Pre}(\mathbb{X}) = \{x \in \mathbb{X} | Ax \in \mathbb{X}\}. \tag{2}$$

Likewise, the (1-)forward reachable set of $\mathbb{X}$ is:

$$\mathrm{Suc}(\mathbb{X}) = \{Ax | x \in \mathbb{X}\}. \tag{3}$$

cf. file *PreReachAut.m*

Now classical discrete-time LTI system:

$$x(k+1) = Ax(k) + BU(k), \tag{4}$$

with constraints $x(k) \in \mathbb{X}, u(k) \in \mathbb{U}, \forall k$.

The (1-)backward reachable set of $\mathbb{X}$ is:

$$\mathrm{Pre}(\mathbb{X}) = \{x \in \mathbb{X} | \exists u \in \mathbb{U} \text{ s.t. } Ax + Bu \in \mathbb{X}\}. \tag{5}$$

Likewise, the (1-)forward reachable set of $\mathbb{X}$ is:

$$\mathrm{Suc}(\mathbb{X}) = \{Ax + Bu | x \in \mathbb{X}, u \in \mathbb{U}\}. \tag{6}$$

cf. file *PreReach.m* and eq. (105) p.75

Definition (9.1 in LN) of a positively invariant set $\mathcal{S}$:

$$x(0) \in \mathcal{S} \;=>\; x(k) \in \mathcal{S}, \quad \forall k. \tag{7}$$

cf. file *Oinf.m*

Definition (9.3 in LN) of a control invariant set $\mathcal{C}$:

$$x \in \mathcal{C} \;\; => \;\; \exists u \in \mathbb{U} \text{ s.t. } Ax + Bu \in \mathcal{C}, \quad \forall k. \tag{8}$$

The maximum control invariant set in $\mathbb{X}$ is noted $\mathcal{C}_\infty$.

Very important notion for recursive feasibility of RHC !

cf. file *Cinf.m*

Question 1: Compute $\mathcal{C}_\infty$ using the recursion proposed in (104) p.75 of LN (i.e. do NOT use the invariantSet() command).

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Case of a RHC with stage constraints $\mathbb{X}$, $\mathbb{U}$, and terminal constraints $x(N) \in \mathbb{X}_f$.

The feasible set $\mathcal{X}_N$ can be computed as the N-step controllable set (cf. Definition 9.5 p.76 of the LN).

cf. file *Cinf.m*

Question 2: Find the smallest horizon length N which gives the maximum controllable set (i.e. the largest feasible set $\mathcal{X}_N$). Here, we assume $\mathbb{X}_f = 0$.