SSY281 PSS 6 - MPT Part 2: MPC and Minimum-Time Control

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March 5^{th} 2021

PSS 6 - MPC with MPT

Design a MPC with the MPT toolbox, cf file *MPC_controller.m*.

How should should $\mathbb{X}_f$ and V_f be chosen? One option is:

Theorem 10.10 (Stability of constrained linear quadratic MPC). *Consider the linear quadratic MPC with linear constraints applied to the controllable system $x^+ = Ax + Bu$ and with positive definite matrices Q and R. Further assume that the terminal cost V_f is chosen as the value function of the corresponding unconstrained, infinite horizon LQ controller, and that the terminal constraint set $\mathbb{X}_f$ is chosen as described above. Then the origin is asymptotically stable with a region of attraction $\mathcal{X}_N$ for the controlled system $x^+ = Ax + B\kappa_N(x)$.*

where $\mathbb{X}_f$ is the LQR invariant set (i.e. (i) no $\mathbb{X}$ or $\mathbb{U}$ constraints are active (ii) control invariant for the infinite horizon LQ control law).

But other choices of $\mathbb{X}_f$ are possible too: $\mathcal{C}_\infty$, $\{0\}$, ...

Reminder:

- Remember the batch approach from chapter 3 of the LN? (page 25)

$$x = \Omega x(0) + \Gamma u.$$

- Take some x (or $x(0)$) in the feasible set.
- The LQ MPC can be written as (page 92):

$$\begin{aligned} \min_{u} \quad & V_N(x, u) = \frac{1}{2}(u^\top \tilde{R} u + x^\top \tilde{Q} x) + u^\top S x \\ \text{subject to} \quad & F u \leq G x + h \end{aligned}$$

  which is a parametric optimization problem with parameter x.
- At the optimal solution u^*, some constraints are active and some are inactive (cf chapter on Optimization in LN).

Key idea: The active set at the optimal solution for x remains the same in a neighborhood of x.

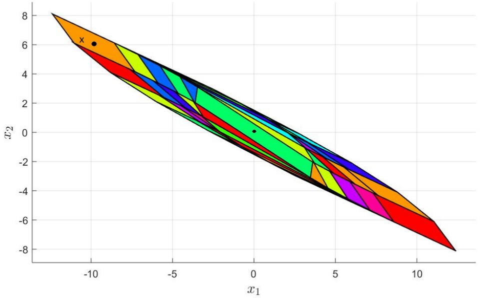

Figure: Critical regions of the feasible set.

- Linear feedback in each region R_i since active set is fixed.
- Off-line phase: compute feedback law in each R_i.
- On-line phase: find current R_i and apply relevant feedback.

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Algorithm from the book: F Borrelli, A. Bemporad, and M. Morari. *Predictive Control for Linear and Hybrid Systems* (Chapter 11, section 5).

Needed for Assignment 8.

- Minimum time control problem:

$$
\begin{aligned}
J_0^*(x(0)) = \quad & \min_{U_0, N} \quad && N \\
& \text{subj. to} && x_{k+1} = Ax_k + Bu_k, \ k = 0, \ldots, N-1 \\
& && x_k \in \mathcal{X}, \ u_k \in \mathcal{U}, \ k = 0, \ldots, N-1 \\
& && x_N \in \mathcal{X}_f \\
& && x_0 = x(0),
\end{aligned}
$$

- Say we want an explicit control law to solve it.
- Go for 1 N-horizon EMPC, or N 1-horizon EMPC?

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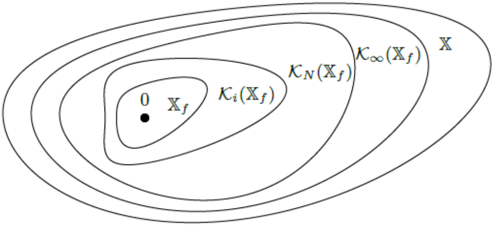
- N 1-horizon EMPC of the form:

$$
\begin{array}{ll}
\min\limits_{u_0} & c(x_0, u_0) \\
\text{subj. to} & x_1 = Ax_0 + Bu_0 \\
& x_0 \in \mathcal{X},\ u_0 \in \mathcal{U} \\
& x_1 \in \mathcal{K}_{j-1}(\mathcal{X}_f),
\end{array}
$$

- By definition, feasible set of each subproblem is $\mathcal{K}_j(\mathbb{X}_f)$.
- Remember:



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Algorithm 1: Minimum-time control algorithm.

Input: x_0
Off-line: solve each subproblem j, starting from $\mathbb{X}_f$.
$x = x_0$
On-line:
while $x \notin X_f$ **do**
    Find smallest controller j s.t. $x \in \mathcal{K}_j(\mathbb{X}_f)$.
    Find controller region $R_{j,i}$ s.t. $x \in R_{j,i}$.
    Apply the corresponding control feedback, i.e. $x = (A + BK_{j,i})x$.
end

MPT function tips:

- *envelope*: Translate a collection of sets (e.g. a EMPC partition..) into a set object.
- *contains*: Check if a point in a given set.
- *evaluate*: Get command from an EMPC controller.