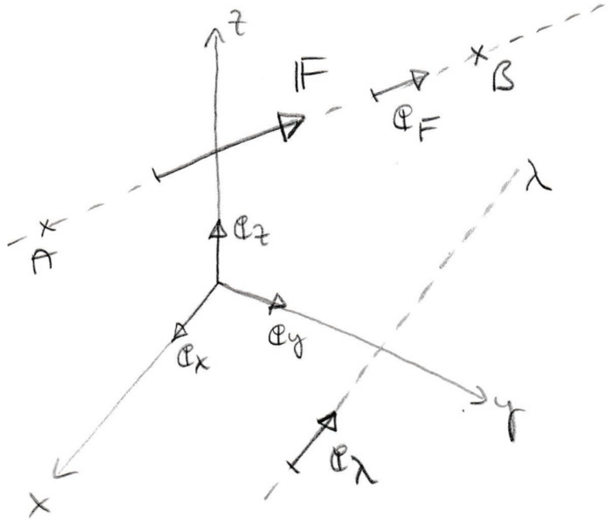
3D kraftsystem (Vektorformalism)

① Krafter i 3D

$\boxed{\text{Cartesiska koordinatsystem}}$ {obs! Högerorient. koord. syst.!}



$$\mathbb{F} = F_x \mathbb{e}_x + F_y \mathbb{e}_y + F_z \mathbb{e}_z =$$

$$= F \mathbb{e}_F$$

$$F = |\mathbb{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$\mathbb{e}_F = \mathbb{e}_{AB} = \frac{\overline{AB}}{|\overline{AB}|}$$

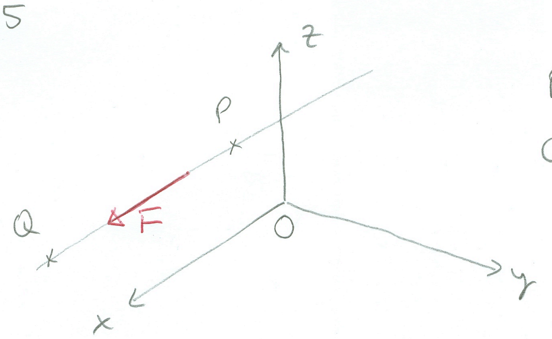
Projektionen av $\mathbb{F}$ på axeln λ (dvs. på riktningen $\mathbb{e}_\lambda$) ges av:

$$\mathbb{F}_\lambda = F_\lambda \mathbb{e}_\lambda \quad \text{där} \quad \boxed{F_\lambda = \mathbb{F} \cdot \mathbb{e}_\lambda}$$

{Anm.: $\mathbb{F}$ och λ behöver ej skära varandra — "korsande linjer"}

Räkna!

1.35



P : (L; −L; 3L)

Q : (2L; −4L; −L)

$$\mathbb{F} = F\, e_{PQ} = F\, \frac{\overline{PQ}}{|PQ|} = F\, \frac{\overline{OQ} - \overline{OP}}{|\overline{OQ} - \overline{OP}|} =$$

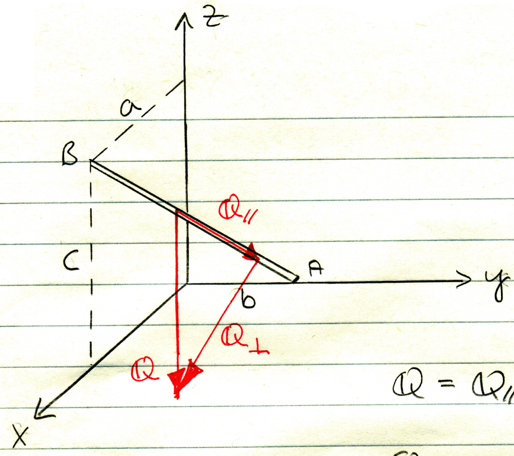
$$= F\, \frac{(2L - L)\, e_x + (-4L - (-L))\, e_y + (-L - 3L)\, e_z}{(L^2 + (3L)^2 + (4L)^2)^{1/2}} =$$

$$= F\, \frac{L\, e_x - 3L\, e_y - 4L\, e_z}{L\,(1 + 9 + 16)^{1/2}} =$$

$$= \frac{F}{\sqrt{26}}\,(e_x - 3\,e_y - 4\,e_z) = F_x e_x + F_y e_y + F_z e_z$$

där $F_y = -\frac{3}{\sqrt{26}} F$

1.36



$$\begin{cases} m = 11{,}8 \text{ kg} \\ b = 0{,}325 \text{ m} \\ c = 0{,}910 \text{ m} \\ a = 0{,}690 \text{ m} \end{cases}$$

$$\mathbb{Q} = \mathbb{Q}_{\parallel} + \mathbb{Q}_{\perp}$$

$$\mathbb{Q} = -mg\, \mathbb{e}_z$$

$$\mathbb{e}_{\parallel} = \mathbb{e}_{BA} = \frac{\overline{BA}}{|\overline{BA}|} = \frac{-a\,\mathbb{e}_x + b\,\mathbb{e}_y - c\,\mathbb{e}_z}{(a^2 + b^2 + c^2)^{1/2}} \equiv$$

$$\equiv -\alpha\, \mathbb{e}_x + \beta\, \mathbb{e}_y - \gamma\, \mathbb{e}_z$$

$\mathbb{Q}_{\parallel} = Q_{\parallel}\, \mathbb{e}_{\parallel}$ dār

$$Q_{\parallel} = \mathbb{Q} \cdot \mathbb{e}_{\parallel} = -mg \cdot (-\gamma) \approx \underline{\underline{88{,}7 \text{ N}}}$$

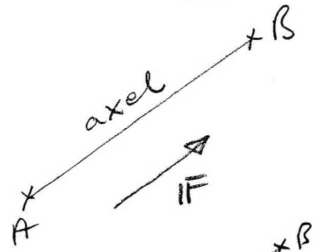
$$Q_{\parallel}^2 + Q_{\perp}^2 = Q^2 \Rightarrow$$

$$Q_{\perp} = (Q^2 - Q_{\parallel}^2)^{1/2} \approx \underline{\underline{74{,}4 \text{ N}}}$$

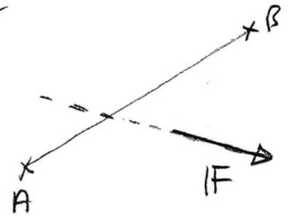
■

② Kraftmomentvektorn

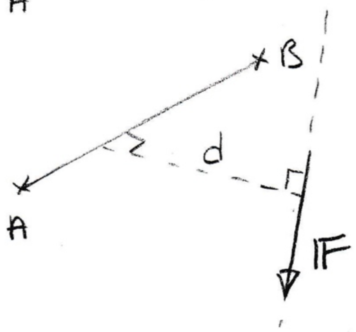
Betrakta först några spec. fall: (som svarar på frågan: Vrider kraften kring axeln?)



$\mathbf{F} \parallel AB$: $M_{AB} = 0$

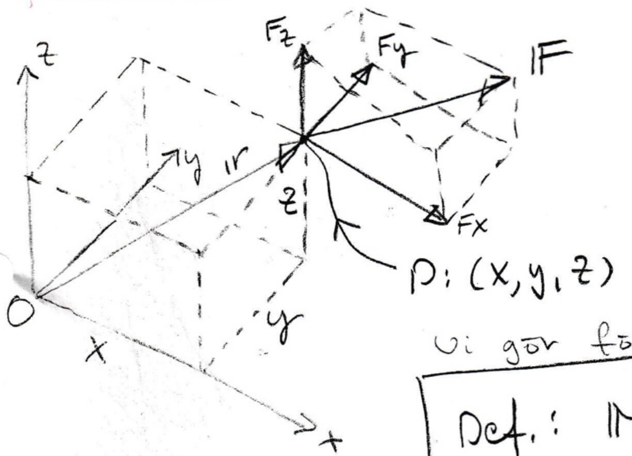


$\mathbf{F}$ skär AB : $M_{AB} = 0$



$\mathbf{F} \perp AB$: $M_{AB} = Fd$

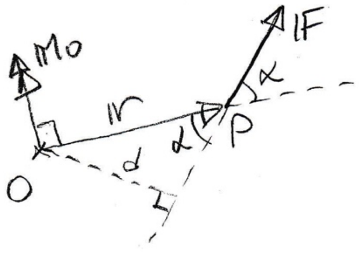
Allmänt:



$$\begin{cases} M_x = F_z y - F_y z \\ M_y = F_x z - F_z x \\ M_z = F_y x - F_x y \end{cases}$$

Vi gör följande

$$\boxed{\text{Def.: } \mathbf{M}_o = \mathbf{r} \times \mathbf{F} = \overline{OP} \times \mathbf{F}}$$

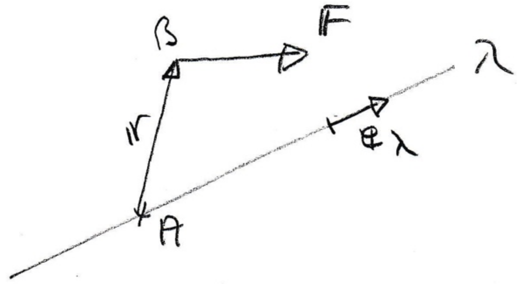


$$|\mathbf{M}_o| = |\mathbf{r}| \cdot |\mathbf{F}| \cdot \sin\alpha = |\mathbf{F}| \cdot d$$

$$\mathbf{M}_o \cdot \mathbf{e}_x = (\mathbf{M}_o)_x = M_x \quad \text{osv.}$$

Ex:

Givet: $\mathbb{F}$ och λ. Sök: M_λ (dvs. kraftens mom. map. linjen λ)



1) Välj A på λ

2) Bilda $\mathbb{M}_A = \mathbb{r} \times \mathbb{F} =$

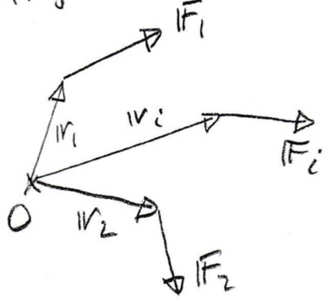
$= \overline{AB} \times \mathbb{F}$

3) M_λ fås enl.:

$$M_\lambda = \mathbb{M}_A \cdot \mathbb{e}_\lambda$$

③ Kraftsumma, momentsumma

Kraftsyst.: $\{\mathbb{F}_1, \mathbb{F}_2, \ldots, \mathbb{F}_n\}$

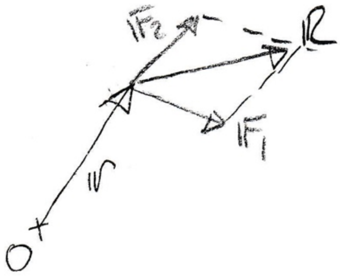


$$\sum \mathbb{F} = \sum_{i=1}^{n} \mathbb{F}_i$$

$$\sum \mathbb{M}_O = \sum_{i=1}^{n} \mathbb{r}_i \times \mathbb{F}_i$$

{ "Varignons teorem" ; Om $\mathbb{R} = \mathbb{F}_1 + \mathbb{F}_2$ så

är $\mathbb{M}_O = \mathbb{r} \times \mathbb{F}_1 + \mathbb{r} \times \mathbb{F}_2 = \mathbb{r} \times (\mathbb{F}_1 + \mathbb{F}_2) = \mathbb{r} \times \mathbb{R}$

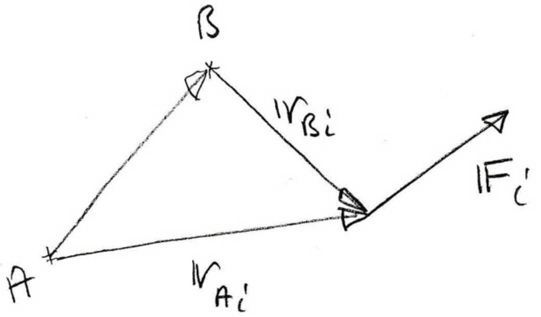


s.k. }

Ex:

A och B två skilda pkter,

Samband mellan $\sum \mathbb{M}_A$ och $\sum \mathbb{M}_B$?



$$\sum \mathbb{M}_A = \sum_{i=1}^{n} \mathbb{r}_{Ai} \times \mathbb{F}_i$$

$$\sum \mathbb{M}_B = \sum_{i=1}^{n} \mathbb{r}_{Bi} \times \mathbb{F}_i$$

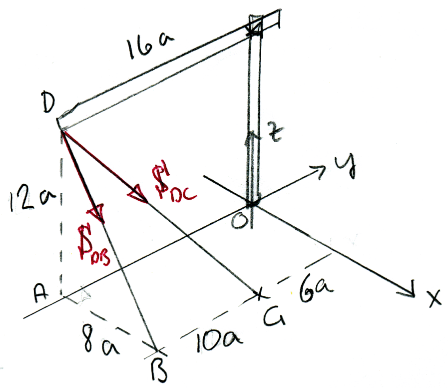
$$\sum \mathbb{M}_A - \sum \mathbb{M}_B = \sum_{i=1}^{n} (\mathbb{r}_{Ai} - \mathbb{r}_{Bi}) \times \mathbb{F}_i =$$

$$= \sum_{i=1}^{n} \overline{AB} \times \mathbb{F}_i =$$

$$= \overline{AB} \times \sum_{i=1}^{n} \mathbb{F}_i = \overline{AB} \times \sum \mathbb{F}$$

$$\therefore \boxed{\sum \mathbb{M}_A = \sum \mathbb{M}_B + \overline{AB} \times \sum \mathbb{F}}$$

1.46



$$\begin{cases} a = 1{,}0\,m \\ S' = 17\,kN \end{cases}$$

(b)

$$\begin{cases} e_{DB} = \dfrac{DB}{|\overline{DB}|} = \dfrac{8a\,e_x - 12a\,e_z}{a(64+1.44)^{1/2}} = \dfrac{1}{\sqrt{208}}(8e_x - 12e_z) \\ S'_{DB} = S'\,e_{DB} = \dfrac{S'}{\sqrt{208}}(8e_x - 12e_z) \\ \overline{OD} = -16a\,e_y + 12a\,e_z \end{cases}$$

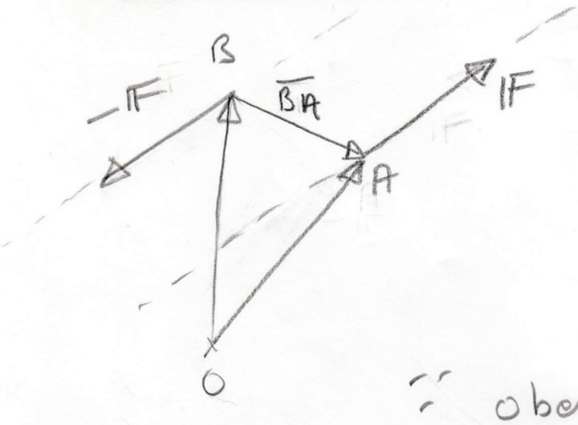
$$\Rightarrow \quad M_O = \overline{OD} \times S'_{DB} = \begin{vmatrix} e_x & e_y & e_z \\ 0 & -16a & 12a \\ \frac{8S'}{\sqrt{208}} & 0 & \frac{-12S'}{\sqrt{208}} \end{vmatrix} =$$

$$= \frac{S'a}{\sqrt{208}}\left\{ e_x(16 \cdot 12) - e_y(-12 \cdot 8) + e_z(16 \cdot 8) \right\} =$$

$$= \frac{S'a}{2\sqrt{52}} \cdot \cancel{16}^{\,8} \left\{ 12e_x + 6e_y + 8e_z \right\} = \frac{S'a \cdot 8 \cdot \cancel{2}}{\cancel{2} \cdot \sqrt{13}}(6,3,4)$$

$$\approx (226, 113, 151)\ kNm$$

④ Kraftpar, rent moment

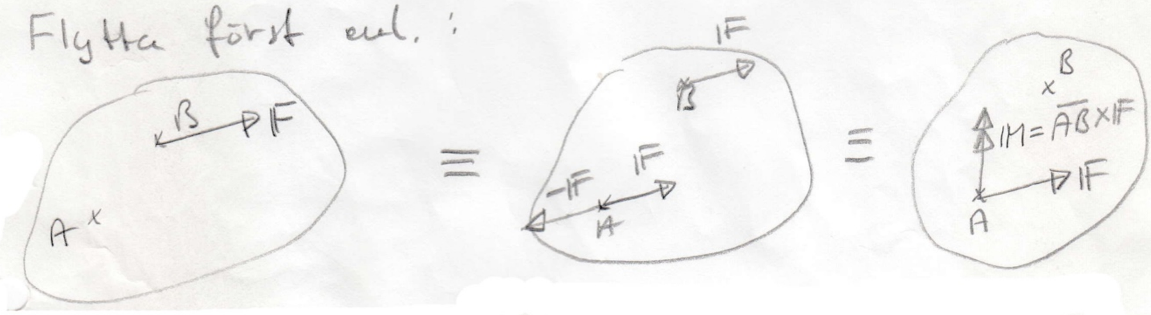


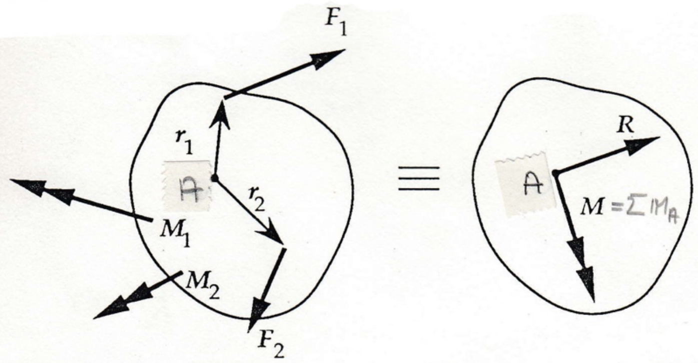
$$\sum \mathbf{M}_O = \overline{OA} \times \mathbf{F} + \overline{OB} \times (-\mathbf{F})$$
$$= (\overline{OA} - \overline{OB}) \times \mathbf{F}$$
$$= \overline{BA} \times \mathbf{F}$$

$\therefore$ ober. av O:s läge!

⑤ Reduktion av kraftsystem (jfr. 2D)

Flytta först en!:





Kraftsystemet $\{F_1,\dots,F_n;M_1,\dots,M_k\}$ kan reduceras till en kraft R angripande i A och ett rent moment M. Det gäller då att $\sum \mathbf{F}$ och $\sum \mathbf{M}_A$ är oförändrade.

$R = \sum_{i=1}^{n} F_i = \Sigma F$, $\quad M = \sum_{i=1}^{n} r_i \times F_i + \sum_{j=1}^{k} M_j$, där r_i är vektorn från A till F_i:s angreppspunkt.