University of Gothenburg
Chalmers university of Technology

Mathematical sciences

MSA251 and TMS032 Experimental design and sampling

Examination

Date: March 20 2019

Time: 4 hours

Examiner: Anders Muszta.

On call: Anders Muszta, telephone 070 3868501.

Aids:
- Calculator;
- Course literature in bound format or printed pages from digital format;
- No handwritten notes are allowed in the course literature.
- Highlighting and unmarked post-it notes are allowed in the course literature.
- Course literature are the two books *Sampling Techniques* by William G. Cochran and *Design and Analysis of Experiments* by Douglas C. Montgomery.

Grading:
- The exam comprises five problem-sets that are awarded at most 33 points.
- Solutions should be legible and easy to follow.
- Solutions that are illegible run the risk of not being assessed.
- Notation that is introduced in the solution process must be accompanied with a definition.

TMS032: Grade 3 requires 40 percent of maximum possible points on the exam and grade 4 requires 60 percent and grade 5 requires 80 percent.

MSA251: Grade G requires 40 percent of maximum possible points on the exam and grade VG requires 65 percent.

Problem 1 (5 points)

a) Give a positive and a negative aspect with employing fractional factorial design. (2p)

b) Give a positive and negative aspect with employing factorial design. (2p)

c) Why should one examine statistical properties of residuals when conducting a regression analysis? (1p)

Problem 2 (9 points)

For each of the following contexts suggest a possible sampling design as well as an appropriate statistical method of analysis and motivate your choices. Each sub-problem can be awarded 3 points.

a) In a forest you investigate the effects of tree species and ground preparation on tree biomass. Parts of the forest grow around a lake.

b) Is there a connection between height and diameter of spruce trees in a forest? The forest grows on fairly even terrain with little topographical variation.

c) You are interested in studying all forests in Sweden and want to sample the forests appropriately due to restricted resources. You are particularly interested in whether the connection between height and diameter of trees in swedish forests depend on tree species?

Problem 3 (6 points)

Let $\{X_k\}_{k=1}^n$ denote independent observations of a random variable that is assumed to be uniformly distributed on some interval $[\theta, 1]$. The objective is to use the observations to estimate parameter θ with estimator $\hat{\theta}_n$ defined as

$$\hat{\theta}_n = \min_{1 \le k \le n} X_k \ .$$

a) (2p) Show that the probability density function of estimator $\hat{\theta}_n$ is

$$\frac{n}{(1-\theta)^n}(1-x)^{n-1} \ , \quad \theta \le x \le 1.$$

b) Show that the bias of $\hat{\theta}_n$ is $\frac{1-\theta}{n+1}$. (1p)

c) Use $\hat{\theta}_n$ to create an unbiased estimator of θ. (1p)

d) Determine the mean square error of $\hat{\theta}_n$. (1p)

e) Show that estimator $\hat{\theta}_n$ is weakly consistent. (1p)

Problem 4 (6 points)

The following effects were estimated in a 2^4 factorial design.

Tabell 1: Estimated effects in a 2^4 factorial design.

Effect	A	B	C	D	AB	AC	AD	BC
Estimate	0.83	4.31	7.94	2.12	0.21	0.75	0.65	1.76
Effect	BD	CD	ABC	ABD	ACD	BCD	ABCD	Mean
Estimate	0.34	3.52	0.13	0.43	0.52	0.11	0.62	7.14

a) Which effects appear to be significant upon visual inspection? (1p)

b) Perform a statistical analysis to judge the significance of the effects, based on the assumption that interaction effects of order three and higher are negligible. Use the level of significance 0.05. (3p)

c) Describe the result of your analysis in a mathematical formula, using sufficiently many factors. (1p)

d) Use the mathematical formula to obtain a prognosis of the response variable when factors A and B are set to a high setting and factors C and D set to a low setting. (1p)

Problem 5 (7 points)

Consider constructing a 2^{7-3} fractional factorial design such that main effect A is not aliased with interaction effect AB.

a) Which are the generators of your design? (1p)

b) Which are the words in your defining relation? (1p)

c) What is the resolution of your design? (1p)

d) Which effects are aliased with the main effect B? (1p) Is your conclusion consistent with the resolution of your design? (1p)

e) Which effects are aliased with the interaction effect AB? (1p) Is your conclusion consistent with the resolution of your design? (1p)
