W5 – integral tillämpningar

- vi fortsätter med integrations tekniker och räknar längder, ytor, och volymer av komplicerade geometriska objekt
- kommentarer i grått eller grafer, skall inte finnas tillgängliga på tentan. de finns nu för att underlätta övningen.

ytan mellan två grafer

$$s = \int_{x=a}^{x=b} ds$$

$$ds = \sqrt{1 + (f'(x))^2}\, dx$$

EXERCISES 5.7

In Exercises 1–16, sketch and find the area of the plane region bounded by the given curves.

1. $y = x, \quad y = x^2$

2. $y = \sqrt{x}, \quad y = x^2$

3. $y = x^2 - 5, \quad y = 3 - x^2$

4. $y = x^2 - 2x, \quad y = 6x - x^2$

27. Find the total area enclosed by the curve $y^2 = x^2 - x^4$

5.7 areas of plane regions

Find the areas of the regions described in Exercises 17–28. It is helpful to sketch the regions before writing an integral to represent the area.

17. Bounded by $y = \sin x$ and $y = \cos x$, and between two consecutive intersections of these curves

18. Bounded by $y = \sin^2 x$ and $y = 1$, and between two consecutive intersections of these curves

AD/7.3 räkna längder av kurvor

In Exercises 1–16, find the lengths of the given curves.

1. $y = 2x - 1$ from $x = 1$ to $x = 3$

2. $y = ax + b$ from $x = A$ to $x = B$

5. $y^3 = x^2$ from $(-1, 1)$ to $(1, 1)$

7. $y = \dfrac{x^3}{12} + \dfrac{1}{x}$ from $x = 1$ to $x = 4$

8. $y = \dfrac{x^3}{3} + \dfrac{1}{4x}$ from $x = 1$ to $x = 2$

7.1 Volymer av roterande grafer

EXERCISES 7.1

$$V = \pi \int_a^b (f(x))^2 \, dx$$

Find the volume of each solid S in Exercises 1–4 ~~in two ways, using the method of slicing and the method of cylindrical shells~~.

1. S is generated by rotating about the x-axis the region bounded by $y = x^2$, $y = 0$, and $x = 1$.

3. S is generated by rotating about the x-axis the region bounded by $y = x^2$ and $y = \sqrt{x}$ between $x = 0$ and $x = 1$.

7.3 ytor av roterande grafer

$$S = 2\pi \int_{x=a}^{x=b} |y|\, ds = 2\pi \int_{a}^{b} |f(x)| \sqrt{1 + (f'(x))^2}\, dx$$

In Exercises 22–29, find the areas of the surfaces obtained by rotating the given curve about the indicated lines.

23. $y = x^3, (0 \leq x \leq 1)$, about the x-axis

26. $y = e^x, (0 \leq x \leq 1)$, about the x-axis

32. **(Area of a prolate spheroid)** Find the area of the surface obtained by rotating the ellipse $x^2 + 4y^2 = 4$ about the x-axis.

7.3 ytor av roterande grafer

39. A hollow container in the shape of an infinitely long horn is generated by rotating the curve $y = 1/x$, $(1 \leq x < \infty)$, about the x-axis.

(a) Find the volume of the container.

(b) Show that the container has infinite surface area.

(c) How do you explain the "paradox" that the container can be filled with a finite volume of paint but requires an infinite amount of paint to cover its surface?