W6 – RÖ differentiella ekvationer

grundläggande tekniker:

- gissa
- fixa homogen och partikulär separat och lägg ihop lösningar
- är ekvationen separabel?
- är den linjär med konstanta koefficienter?
- kommentarer i grått eller grafer, skall inte finnas tillgängliga på tentan. de finns nu för att underlätta övningen.

klasificiera diff ekvationer **EXERCISES 18.1**

In the exercises, state the order of the given DE and whether it is linear or nonlinear. If linear, is it homogeneous or nonhomogeneous? also, specify coefficient functions that match the template: $a_n(x)y^{(n)}(x) + \cdots + a_1(x)y'(x) + a_0(x)y(x) = f(x)$. Indicate if separable.

1. $\dfrac{dy}{dx} = 5y$ $a_1(x) =$ $a_0(x) =$ $f(x) =$

2. $\dfrac{d^2y}{dx^2} + x = y$ $a_2(x) =$ $a_1(x) =$ $a_0(x) =$ $f(x) =$

3. $y\dfrac{dy}{dx} = x$ $a_1(x) =$ $a_0(x) =$ $f(x) =$

4. $y''' + xy' = x \sin x$ $a_3(x) =$ $a_2(x) =$ $a_1(x) =$ $a_0(x) =$ $f(x) =$

5. $y'' + x \sin x\, y' = y$ $a_2(x) =$ $a_1(x) =$ $a_0(x) =$

6. $y'' + 4y' - 3y = 2\boxed{y^2}$ $a_2(x) =$ $a_1(x) =$ $a_0(x) =$

7. $\dfrac{d^3y}{dt^3} + t\dfrac{dy}{dt} + t^2 y = t^3$

$a_3(t) =$ $a_2(t) =$ $a_1(t) =$ $a_0(t) =$ $f(t) =$

8. $\boxed{\cos x}\dfrac{dx}{dt} + x \sin t = 0$ $a_2(x) =$ $a_1(x) =$ $a_0(x) =$

9. $y^{(4)} + e^x y'' = x^3 y'$

$a_4(x) =$ $a_3(x) =$ $a_2(x) =$ $a_1(x) =$ $a_0(x) =$

10. $x^2 y'' + e^x y' = \boxed{\dfrac{1}{y}}$ $a_2(x) =$ $a_1(x) =$ $a_0(x) =$

$f(t) =$

- H = homogen
- IH = icke homogen
- L = linjär
- IL = icke linjär
- G=n; av grad n
- LKK = linjär med konstanta koefficienter
- S=separabel
- IS=icke separabel

ADAMS/7.9 separable equations

EXERCISES 7.9

Solve the separable equations in Exercises 1–10.

1. $\dfrac{dy}{dx} = \dfrac{y}{2x}$

3. $\dfrac{dy}{dx} = \dfrac{x^2}{y^2}$

$$\frac{dy}{dx} = f(x)g(y) \rightarrow \frac{dy}{g(y)} = f(x)dx \rightarrow \int \frac{dy}{g(y)} = \int f(x)dx$$

Stenmark, bara att göra

7. $\dfrac{dy}{dx} = 1 - y^2$

2. $\dfrac{dy}{dx} = \dfrac{3y-1}{x}$

lite svårare; det gäller att inse att
(7) $f(x)=1$ och $g(y)=1-y^2$
(2) $f(x)=1/x$ och $g(y)=3y-1$

ADAMS/7.9 linjära ekvationer med icke konstanta koefficient funktioner

EXERCISES 7.9 Solve the linear equations

11. $\frac{dy}{dx} - \frac{2y}{x} = x^2$

12. $\frac{dy}{dx} + \frac{2y}{x} = \frac{1}{x^2}$

18. $\begin{cases} \frac{dy}{dx} + 3x^2 y = x^2 \\ y(0) = 1 \end{cases}$

$y'(x) + p(x)y(x) = q(x)$

(1) $\mu(x) = \int p(x)\,dx$

(2) $A(x) = \int e^{\mu(x)} q(x) dx$

(3) $y(x) = A(x)e^{-\mu(x)}$

(11)	(12)	(18)
$p(x) =$	$p(x) =$	$p(x) =$
$q(x) =$	$q(x) =$	$q(x) =$
$\mu(x) =$	$\mu(x) =$	$\mu(x) =$
$A(x) =$	$A(x) =$	$A(x) =$
$y(x) =$	$y(x) =$	$y(x) =$

linjära DE med konstanta koefficient funktioner

EXERCISES 3.7 $e^{iu} = \cos u + i \sin u$ (EU)

VG

13. $\begin{cases} 2y'' + 5y' - 3y = 0 \\ y(0) = 1 \\ y'(0) = 0. \end{cases}$

Svar: $y(x) = \frac{1}{7} e^{-3x}\left(1 + 6e^{7x/2}\right)$

VG

27. Solve $\begin{cases} y'' + y = 0 \\ y(2) = 3 \\ y'(2) = -4. \end{cases}$

EU

Svar: $y(x) = 3\cos(2 - x) - 4\sin(2 - x)$

VG

33. $\begin{cases} y'' + 2y' + 5y = 0 \\ y(3) = 2 \\ y'(3) = 0. \end{cases}$

EU

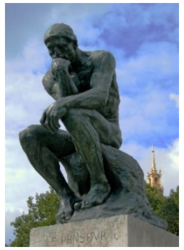
Svar: $y(x) = e^{3-x}\,[2\cos(6-2x) - \sin(6-2x)]$

MVG **EXERCISES 18.5**

5. Show that $y = e^{2t}$ is a solution of

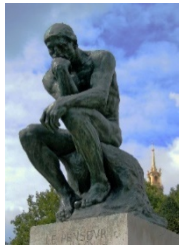
$$y''' - 2y' - 4y = 0$$

and find the general solution of this DE.



MVG

6. Write the general solution of the linear, constant-coefficient DE having auxiliary equation $(r^2 - r - 2)^2(r^2 - 4)^2 = 0$.



ett problem för ”de som vågar”

MVG^2

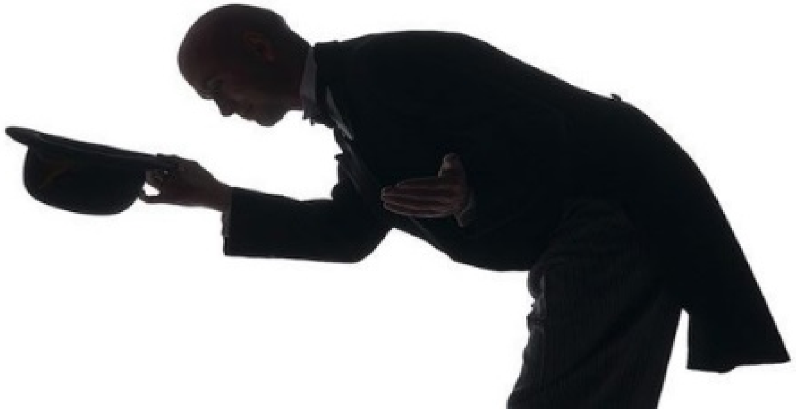
Differential equation of an order higher than 2 is given in an operator form

$$\left(4-\frac{d^2}{dx^2}\right)\left(1+\frac{d}{dx}\right)^3 y(x)=0$$

(1) How does the equation look like in its normal form?

(2) Write down the characteristic equation for the differential equation.

(3) Write the general solution of the equation.



Hint: What is the result of applying the operator $D=a+\frac{d}{dx}$ on the function $f(x)=e^{-ax}u(x)$? Calculate $Df(x)=\left(a+\frac{d}{dx}\right)f(x)=\left(a+\frac{d}{dx}\right)\left(e^{-ax}u(x)\right)=ae^{-ax}u(x)+\frac{d}{dx}\left(e^{-ax}u(x)\right)$ and see what happens. This should give you a very clear idea what happens when one applies D twice, or when it is applied any number of times.