Mathematics Chalmers & GU

MVE455: Partial Differential Equations, 2020–03–16, 8:30-12:30

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Calculators, formula notes and other subject related material are not allowed.
Each problem gives max 4p. Valid bonus poits will be added to the scores.
Breakings: **3**: 10-15p, **4**: 16-20p och **5**: 21p-

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1. Let $\pi_1 f(x)$ be the linear interpolant of a, twice continuously differentiable, function f on the interval $I = (a, b)$. Prove the following *optimal* interpolation error estimate for the first derivative:

$$||f' - (\pi_1 f)'||_{L_\infty(a,b)} \le \frac{1}{2}(b-a)||f''||_{L_\infty(a,b)}$$

2. Consider the Poisson equation with Neumann boundary condition and with $k > 0$:

$$(1) \qquad \begin{cases} -\Delta u = f, & \text{in } \Omega \subset \mathbf{R}^2, \\ -\mathbf{n} \cdot \nabla u = k\, u, & \text{on } \partial\Omega, \end{cases}$$

where $\mathbf{n}$ is the outward unit normal to $\partial\Omega$ (the boundary of Ω). Prove the Poincare inequality:

$$\|u\|_{L_2(\Omega)} \le C_\Omega(\|u\|_{L_2(\partial\Omega)} + \|\nabla u\|_{L_2(\Omega)}).$$

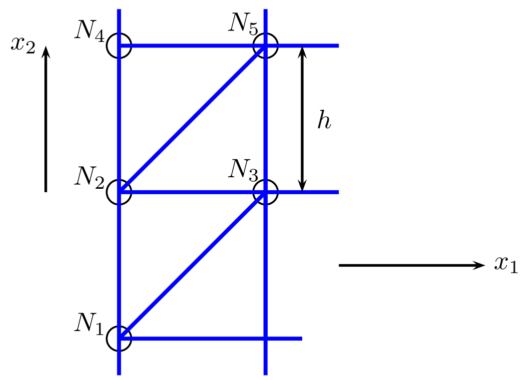
3. a) Let p be a positive constant. Prove an a priori error estimate (in the H^1-norm: $\|e\|_{H^1}^2 = \|e'\|^2 + \|e\|^2$) for the standard cG(1) finite element method for problem

$$-u'' + pxu' + (1 + \frac{p}{2})u = f, \quad \text{in } (0,1), \qquad u(0) = u(1) = 0.$$

b) For which value of p the error estimate is optimal. Any physical interpolation?

4. Let Ω be the domain in the figure below, with the given triangulation and nodes N_i, $i = 1, \ldots, 5$.
a) Dervie mass-matrix S and load vector $\mathbf{b}$ in $SU = \mathbf{b}$ for the cG(1) solution U to the problem

$$\begin{cases} -\Delta u = 1, & \text{in } \Omega \subset \mathbf{R}^2, \\ -\mathbf{n} \cdot \nabla u = 0 & \text{on } \partial\Omega. \end{cases}$$



b) Given the test function φ_4 at node N_4, find an expression for U_4 in terms of U_2 and U_5.

5. Derive the conservation of enegy for the wave equation in higher dimensions.

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