**Solutions to problem 57.13 on coulurings of the vertices of a cube**

There are 24 rotations of the cube. They are (see Example 57.3 in Durbin’s book)

1. The identity.

2. Three 180° rotations around lines joining the centers of opposite faces.

3. Six 90° rotations around lines joining the centers of opposite faces.

4. Six 180° rotations around lines joining the midpoints of opposite edges.

5. Eight 120° rotations around lines joining opposite vertices.

These rotations form a group *G* acting on the set *T* of 2-colourings of the eight vertices. For *g*∈*G*, let Ψ(*g*) be the number of 2-colourings preserved by *g*. It is equal to 2*n*(*g*) for the number *n*(*g*) of orbits of the action of <*g*> on the set *S* of the eight vertices of the cube.

We have for *g* of type 1, 2, 3, 4 resp. 5 the following <*g*>−orbits on *S*.

1. Eight orbits of length 1.

2. Four orbits of length 2.

3. Two orbits of length 4.

4. Four orbits of length 2.

5. Two orbits of length 3 and two orbits of length 1.

In particular, Ψ(*g*)=28 , 24 , 24 , 24 resp. 24 such that *o*(*G*)−1∑g∈*G*Ψ(*g*)= ∑*g*∈*G*2*n*(*g*) =(1×28 +3×24 +6×22 +6×24 +8×24) =

(1×26 +3×22 +6×20 +6×22 +8×22) = =23.

There are thus by Burnside’s lemma 23 inequivalent 2-colourings of the eight vertices.