Tentamen TMA227 / LGMA62 2020-06-05

Lösningsförslag

1) Dimensionen för V ges av antalet element i en bas $\Rightarrow$ $\dim V = 2$

Basbytesmatrisen från B till C ges av

$$\underset{C\leftarrow B}{T} = \left[ [b_1]_C \; [b_2]_C \right] = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$$

Vi har vidare

$$\underset{B\leftarrow C}{T} = \left(\underset{C\leftarrow B}{T}\right)^{-1} = -\frac{1}{3}\begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$$

$$\Rightarrow \quad [F]_C = \underset{C\leftarrow B}{T} \, [F]_B \, \underset{B\leftarrow C}{T}$$

$$= \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -1 & 5 \\ 0 & -3 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -2 & 7 \\ -1 & 8 \end{bmatrix}$$

Vi söker sedan alla $v \in V$ s.a.

$$[F(v)]_c = [F]_c [v]_c = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\Leftrightarrow \left[\begin{array}{cc|c} -\frac{2}{3} & \frac{7}{3} & -1 \\ -\frac{1}{3} & \frac{8}{3} & 1 \end{array}\right] \begin{array}{l} \cdot(-3) \\ \cdot(-3) \end{array} \sim \left[\begin{array}{cc|c} 2 & -7 & 3 \\ 1 & -8 & -3 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 1 & -8 & -3 \\ 2 & -7 & 3 \end{array}\right] \begin{array}{l} -2 \\ \phantom{x} \end{array} \sim \left[\begin{array}{cc|c} 1 & -8 & -3 \\ 0 & 9 & 9 \end{array}\right] \begin{array}{l} \phantom{x} \\ \cdot\frac{1}{9} \end{array}$$

$$\sim \left[\begin{array}{cc|c} 1 & -8 & -3 \\ 0 & 1 & 1 \end{array}\right] \begin{array}{l} \phantom{x} \\ 8 \end{array} \sim \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 1 \end{array}\right]$$

$$\Rightarrow [v]_c = \begin{bmatrix} 5 \\ 1 \end{bmatrix} \in \mathbb{R}^2$$ entydig lösning

$$\Rightarrow v = 5c_1 + c_2 \in V$$

Svar: $\dim V = 2$, $v = 5c_1 + c_2$

2) Vi har att $F(0) = 0 \in \mathbb{P}_2$ samt

$$F(\alpha p + \beta q)(t) = 3(\alpha p(t) + \beta q(t)) - (p(1-t) + q(1-t))$$

$$= \alpha\left(3p(t) - p(1-t)\right) + \beta\left(3q(t) - q(1-t)\right)$$

$$= \alpha F(p)(t) + \beta F(q)(t) \quad \forall p, q \in \mathbb{P}_2, \ \alpha, \beta \in \mathbb{R}$$

$\Rightarrow$ $F$ är linjär avbildning $\boxed{\text{?}}$

För att bestämma egenvärden och egenfunktioner låter vi $\mathcal{B} = \{1, t, t^2\}$ vara standardbasen för $\mathbb{P}_2$

$$F(b_1) = F(1) = 3 - 1 = 2$$

$$F(b_2) = F(t) = 3t - (1-t) = -1 + 4t$$

$$F(b_3) = F(t^2) = 3t^2 - (1-t)^2 = -1 + 2t + 2t^2$$

$$\Rightarrow [F]_{\mathcal{B}} = \left[\, [F(b_1)]_{\mathcal{B}} \ [F(b_2)]_{\mathcal{B}} \ [F(b_3)]_{\mathcal{B}} \,\right]$$

$$= \begin{bmatrix} 2 & -1 & -1 \\ 0 & 4 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$\uparrow$ $[F]_{\mathcal{B}}$ triangulär!

$\Rightarrow$ Egenvärden: $\lambda_1 = 2$ $\quad \lambda_2 = 4$ (diagonalelementen)

mult. 2

<u>$\lambda_1 = 2$:</u> $\left([F]_B - \lambda I_3\right)[u_1]_B = \left([F]_B - 2I_3\right)[u_1]_B = 0$

$$\Leftrightarrow \left[\begin{array}{ccc|c} 0 & -1 & -1 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

fria variabler

$$\Rightarrow [u_1]_B = a\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + b\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \qquad a, b \in \mathbb{R}$$

<u>$\lambda_2 = 4$:</u> $\left([F]_B - \lambda I_3\right)[u_2]_B = \left([F]_B - 4I_3\right)[u_2]_B = 0$

$$\Leftrightarrow \left[\begin{array}{ccc|c} -2 & -1 & -1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & -2 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

fri variabel

$$\Rightarrow [u_2]_B = c\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} \qquad c \in \mathbb{R}$$

$\Rightarrow$ Motsvarande egenfunktioner till $F : \mathbb{P}_2 \to \mathbb{P}_2$

$$u_1(t) = a \cdot 1 + b(-t) + bt^2 = a + b(-t+t^2) \quad a,b \in \mathbb{R}$$

$$u_2(t) = c \cdot (-1) + c \cdot (2t) = c(-1+2t) \quad c \in \mathbb{R}$$

Svar: Egenvärdena till $F$ är $\lambda_1 = 2$ (mult. 2) och $\lambda_2 = 4$ med motsvarande egenfunktioner

$u_1(t) = a + b(-t+t^2)$ $a,b \in \mathbb{R}$ och

$u_2(t) = c(-1+2t)$ $c \in \mathbb{R}$.

3) $U = \text{Span}\{1, t^2\}$ och $\{1, t^2\} \subset \mathbb{P}_2$

$\Rightarrow$ $U$ underrum av $V = \mathbb{P}_2$ $\boxtimes$

Vidare är $\{1, t^2\}$ linjärt oberoende ty

$$\alpha + \beta t^2 = 0 \quad \forall t \iff \alpha = \beta = 0$$

$\Rightarrow$ $B = \{1, t^2\}$ utgör bas för $U$

$$\langle 1, t^2 \rangle = \int_0^1 t^2 \, dt = \left[\tfrac{1}{3}t^3\right]_0^1 = \frac{1}{3} \neq 0$$

dvs. $b_1 = 1$ och $b_2 = t^2$ ej ortogonala!

Gram-Schmidt:

$v_1 = b_1 = 1$ $\qquad \|v_1\|^2 = \langle v_1, v_1 \rangle = \int_0^1 dt = 1$

$$v_2 = b_2 - \frac{\langle b_2, v_1 \rangle}{\langle v_1, v_1 \rangle} v_1$$

$$= t^2 - \frac{\langle t^2, 1 \rangle}{\|v_1\|^2} 1 = t^2 - \frac{1}{3}$$

$$\|v_2\|^2 = \langle v_2, v_2 \rangle = \int_0^1 \left(t^2 - \tfrac{1}{3}\right)^2 dt = \int_0^1 \left(t^4 - \tfrac{2}{3}t^2 + \tfrac{1}{9}\right) dt$$

$$= \left[\tfrac{1}{5}t^5 - \tfrac{2}{9}t^3 + \tfrac{1}{9}t\right]_0^1 = \frac{1}{5} - \frac{2}{9} + \frac{1}{9} = \frac{4}{45}$$

Ortogonalprojektionen av $q(t) = t$ på $U$ ges av

$$\text{proj}_U\, t = \frac{\langle v_1, t\rangle}{\|v_1\|^2} v_1 + \frac{\langle v_2, t\rangle}{\|v_2\|^2} v_2$$

$$= \langle 1, t\rangle + \frac{45}{4}\left(t^2 - \frac{1}{3}\right)\left\langle t^2 - \frac{1}{3}, t\right\rangle$$

$$= \int_0^1 t\,dt + \frac{45}{4}\left(t^2 - \frac{1}{3}\right)\int_0^1 t\left(t^2 - \frac{1}{3}\right)dt$$

$$= \left[\frac{1}{2}t^2\right]_0^1 + \frac{45}{4}\left(t^2 - \frac{1}{3}\right)\int_0^1 \left(t^3 - \frac{1}{3}t\right)dt$$

$$= \frac{1}{2} + \frac{45}{4}\left(t^2 - \frac{1}{3}\right)\underbrace{\left[\frac{1}{4}t^4 - \frac{1}{6}t^2\right]_0^1}_{= \frac{1}{4} - \frac{1}{6} = \frac{1}{12}}$$

$$= \frac{1}{2} + \frac{45}{48}\left(t^2 - \frac{1}{3}\right) = \left(\frac{1}{2} - \frac{15}{48}\right) + \frac{45}{48}t^2$$

$$= \frac{9}{48} + \frac{45}{48}t^2 = \frac{3}{16} + \frac{15}{16}t^2$$

<u>Svar:</u> Ortogonalprojektionen av $q(t) = t$

är $\text{proj}_U\, q(t) = \frac{3}{16} + \frac{15}{16}t^2$

4) Karakt. ekv. : $r^2 - 5 = 0 \Rightarrow r_{1,2} = \pm\sqrt{5}$

$$\Rightarrow y_n^{(h)} = C_1(\sqrt{5})^n + C_2(-\sqrt{5})^n = (C_1 + (-1)^n C_2)\, 5^{n/2}$$

Linjäritet $\Rightarrow y_n^{(p)} = y_n^{(p_1)} + y_n^{(p_2)}$ där

$$y_{n+2}^{(p_1)} - 5y_n^{(p_1)} = 2^n, \quad y_{n+2}^{(p_2)} - 5y_n^{(p_2)} = -n \qquad (*)$$

$\Rightarrow$ Ansätt partikulärlösningar :

$$y_n^{(p_1)} = A \cdot 2^n, \quad y_n^{(p_2)} = Bn + C$$

Insättning i $(*)$ ger då

1) $A\,2^{n+2} - 5A\,2^n = (4A - 5A)\,2^n = -A\,2^n = 2^n$

$$\Rightarrow A = -1 \Rightarrow y_n^{(p_1)} = -2^n$$

2) $(B(n+2) + C) - 5(Bn + C) = -4Bn + (2B - 4C) = -n$

$$\Rightarrow \begin{cases} -4B = -1 \\ 2B - 4C = 0 \end{cases} \Rightarrow \begin{cases} B = \frac{1}{4} \\ C = \frac{1}{2}B = \frac{1}{8} \end{cases}$$

$$\Rightarrow y_n^{(p_2)} = \frac{1}{4}n + \frac{1}{8} = \frac{1}{8}(2n+1)$$

Alltså ges allmän lösning till differensekv. av

$$y_n = y_n^{(h)} + y_n^{(p_1)} + y_n^{(p_2)}$$

$$= \left(C_1 + (-1)^n C_2\right) 5^{n/2} - 2^n + \frac{1}{8}(2n+1)$$

Begynnelsevillkoren ger:

$$\begin{cases} y_0 = C_1 + C_2 - 1 + \frac{1}{8} = C_1 + C_2 - \frac{7}{8} = -\frac{7}{8} \\ y_1 = (C_1 - C_2)\sqrt{5} - 2 + \frac{3}{8} = \frac{3}{8} \end{cases}$$

$$\Leftrightarrow \begin{cases} C_1 + C_2 = 0 \\ C_1 - C_2 = \frac{2}{\sqrt{5}} \end{cases} \Rightarrow \begin{cases} C_1 = \frac{1}{\sqrt{5}} \\ C_2 = -\frac{1}{\sqrt{5}} \end{cases}$$

$$\Rightarrow y_n = \left(5^{-1/2} - 5^{-1/2}(-1)^n\right) 5^{n/2} - 2^n + \frac{1}{8}(2n+1)$$

$$= \left(1 + (-1)^{n+1}\right) 5^{\frac{n-1}{2}} - 2^n + \frac{1}{8}(2n+1)$$

<u>Svar</u>: $y_n = \left(1 + (-1)^{n+1}\right) \cdot 5^{\frac{n-1}{2}} - 2^n + \frac{1}{8}(2n+1)$

5 a) Vi har $\sum_{k=1}^{\infty} a_k$ med $a_k = \ln\left(\frac{k}{k+4}\right)$

$$a_k = \ln\left(\frac{k}{k+4}\right) = \ln\left(\frac{1}{1+\frac{4}{k}}\right) = -\ln\left(1+\frac{4}{k}\right)$$

$$= \left\{ \ln(1+x) = x + O(x^2) \text{ då } x \to \infty \right\}$$

$$= -\left(\frac{4}{k} + O\left(\frac{1}{k^2}\right)\right)$$

Vi noterar att $a_k < 0 \quad \forall k \in \mathbb{N}$

Studera istället <u>positiva</u> serien $\sum_{k=1}^{\infty} b_k$ med $b_k = -a_k$

$\Rightarrow \quad b_k = \frac{4}{k} + O\left(\frac{1}{k^2}\right) \qquad$ Låt $c_k = \frac{1}{k}$

$\Rightarrow \quad \frac{b_k}{c_k} = 4 + O\left(\frac{1}{k}\right) \xrightarrow[k \to \infty]{} 4$

JK på gränsvärdesform $\Rightarrow \sum_{k=1}^{\infty} b_k$ divergent

ty $\sum_{k=1}^{\infty} \frac{1}{k}$ divergent

$\Rightarrow \sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} (-1) b_k$ divergent

<u>Svar</u>: $\sum_{k=1}^{\infty} \ln\left(\frac{k}{k+4}\right)$ divergerar.

b) Vi har $\sum_{k=1}^{\infty} (-1)^k a_k$ med $a_k = \sin\left(\frac{\pi}{k}\right)$

Vi noterar att $a_k$ är avtagande

för $k \geq 2$ samt $a_k = \sin\left(\frac{\pi}{k}\right) \xrightarrow[k \to \infty]{} 0$

$$\overset{\text{Leibniz}}{\Longrightarrow} \sum_{k=2}^{\infty} (-1)^k a_k \text{ konvergent}$$

$$\Rightarrow \sum_{k=1}^{\infty} (-1)^k a_k \text{ konvergent}$$

<u>Svar</u>: $\sum_{k=1}^{\infty} (-1)^k \sin\left(\frac{\pi}{k}\right)$ konvergerar.

6) Ansätt potensserie $y(x) = \sum_{n=0}^{\infty} a_n x^n$ med $R>0$

$$xy' = x\sum_{n=1}^{\infty} n a_n x^{n-1} = \sum_{n=1}^{\infty} n a_n x^n = \sum_{n=0}^{\infty} n a_n x^n$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} = \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n$$

$$x^2 y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^n = \sum_{n=0}^{\infty} n(n-1) a_n x^n$$

Insättning i differentialekv. ger

$$(1-x^2)y'' - xy' + p^2 y$$

$$= \sum_{n=0}^{\infty} \left[(n+2)(n-1) a_{n+2} - n(n-1)a_n - n a_n + p^2 a_n\right] x^n \equiv 0$$

$$\Rightarrow \quad (n+2)(n+1) a_{n+2} - n^2 a_n + p^2 a_n = 0 \qquad n = 0, 1, 2, \ldots$$

$$\Leftrightarrow \quad a_{n+2} = \frac{n^2 - p^2}{(n+2)(n+1)} a_n = \frac{(n+p)(n-p)}{(n+2)(n+1)} a_n \quad n = 0, 1, 2 \ldots$$

Begynnelsevillkoren ger

$$y(0) = a_0 = 0 \quad , \quad y'(0) = a_1 = 1$$

$$\Rightarrow \quad a_2 = a_4 = a_6 = \ldots = 0$$

$a_1 = 1$

$$a_3 = -\frac{(p-1)(p+1)}{2\cdot 3} a_1 = -\frac{(p-1)(p+1)}{3!}$$

$$a_5 = -\frac{(p-3)(p+3)}{4\cdot 5} a_3 = \frac{(p-3)(p-1)(p+1)(p+3)}{5!}$$

$\vdots$

$$\Rightarrow \begin{cases} a_{2k} = 0 \\ a_{2k+1} = \dfrac{(-1)^k}{(2k+1)!}(p-2k+1)(p-2k+3)\dots(p+2k-3)(p+2k-1) \end{cases}$$

För att avgöra konvergensradien noterar vi att

för $p = 2m+1$, $m \in \mathbb{Z}$ har vi

$a_{2k+1} = 0$ för $2k+1 > p \Leftrightarrow k > \frac{p-1}{2}$

$$\Rightarrow y(x) = \sum_{k=0}^{m} a_{2k+1} x^{2k+1}$$

som är en ändlig summa $\forall x \in \mathbb{R} \Rightarrow R(p) = \infty$

För $p \neq 2m+1$, $m \in \mathbb{Z}$ har vi

$$y = \sum_{k=0}^{\infty} a_{2k+1} x^{2k+1} = x \sum_{k=0}^{\infty} a_{2k+1} x^{2k}$$

Låt $t = x^2$

$$\Rightarrow \sum_{k=0}^{\infty} a_{2k+1} x^{2k} = \sum_{k=0}^{\infty} a_{2k+1} t^k$$

$$\left| \frac{a_{2(k+1)+1}}{a_{2k+1}} \right| = \left| \frac{a_{2k+3}}{a_{2k+1}} \right| = \frac{(2k+1)^2 - p^2}{(2k+3)(2k+5)}$$

$$= \frac{4k^2 + 4k + 1 - p^2}{4k^2 + 16k + 15} \xrightarrow[k \to \infty]{} 1$$

Kvotformeln ger då konvergensradien 1

för $\sum_{k=0}^{\infty} a_{2k+1} t^k$ $\Rightarrow$ Konvergensradien

för $x \sum_{k=0}^{\infty} a_{2k+1} x^{2k}$ är $R = \sqrt{1} = 1$

<u>Svar:</u> $y(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} (p-2k+1) \cdots (p+2k-1) x^{2k+1}$

med konvergensradie $R(p) = \begin{cases} \infty & p = 2m+1 \quad m \in \mathbb{Z} \\ 1 & p \neq 2m+1 \quad m \in \mathbb{Z} \end{cases}$

7) Med $f_k(x) = \ln x \cdot e^{-x/k}$ har vi

$$f_k(x) \xrightarrow[k\to\infty]{} f(x) = \ln x \quad \forall x \in [1,a]$$

Alltså har vi att

$$|f(x) - f_k(x)| = |\ln x \, e^{-x/k} - \ln x|$$

$$= \ln x \, |e^{-x/k} - 1| = \ln x \, (1 - e^{-x/k}) \text{ på } [1,a]$$

$\Rightarrow |f - f_n|$ <u>växande</u> på $[1,a]$ $\forall k \in \mathbb{N}$

$$\Rightarrow \max_{x\in[1,a]} |f(x) - f_k(x)| = |f(a) - f_k(a)|$$

$$= \ln a \underbrace{\left(1 - e^{-a/k}\right)}_{\xrightarrow[k\to\infty]{} 0} \xrightarrow[k\to\infty]{} 0$$

$\Rightarrow \forall \varepsilon > 0 \ \exists N > 0 : n > N \Rightarrow |f - f_k| < \varepsilon \ \forall x \in [1,a]$

$\therefore f_k \to f$ likformigt på $[1,a]$

vilket betyder att vi kan använda satsen

om gränsövergång under integraltecknet

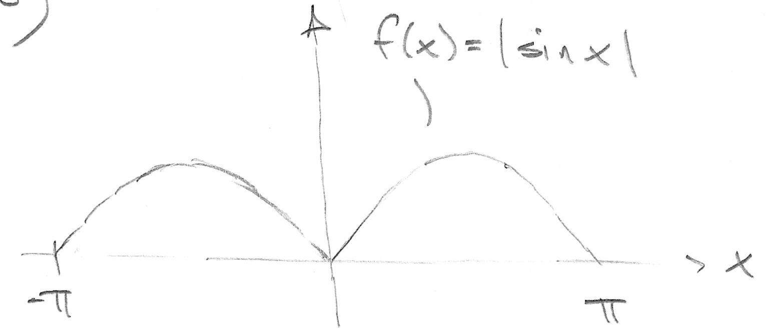
$$\Rightarrow \lim_{k\to\infty} \int_1^a f_k(x)\,dx = \int_1^a f(x)\,dx = \int_1^a \ln x\,dx$$

$$\overset{PI}{=} \left[x \ln x\right]_1^a - \int_1^a dx$$

$$= a \ln a - (a-1) = 1 + a(\ln a - 1)$$

<u>Svar:</u> $\lim_{k\to\infty} \int_1^a \ln(x)\, e^{-x/k} = 1 + a(\ln a - 1)$

8)




$f(x)$ jämn funktion

$\Rightarrow b_k = 0 \quad k = 1, 2, \ldots$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} |\sin x|\,dx = \frac{2}{\pi}\int_0^{\pi} \sin x\,dx = \frac{2}{\pi}\Big[-\cos x\Big]_0^{\pi} = \frac{4}{\pi}$$

$$a_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} |\sin x| \cos x\,dx = \frac{2}{\pi}\int_0^{\pi} \sin x \cos x\,dx$$

$$= \left\{ \sin 2x = 2 \sin x \cos x \right\} = \frac{1}{\pi}\int_0^{\pi} \sin 2x\,dx$$

$$= \frac{1}{\pi}\Big[-\frac{1}{2}\cos 2x\Big]_0^{\pi} = 0$$

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} |\sin x| \cos kx\,dx = \frac{2}{\pi}\int_0^{\pi} \sin x \cos kx\,dx$$

$$= \left\{ \int \sin x \cos kx\,dx = \frac{1}{k^2-1}\left(k \sin x \sin kx + \cos x \cos kx\right) \right\}$$

$$= \frac{2}{\pi}\Big[\frac{1}{k^2-1}\left(k \sin x \sin kx + \cos x \cos kx\right)\Big]_0^{\pi}$$

$$= \frac{2}{\pi}\,\frac{1}{k^2-1}\Big[-\underbrace{\cos \pi k}_{=(-1)^k} - 1\Big] = \frac{2}{\pi}\,\frac{1+(-1)^k}{1-k^2} \qquad k = 2, 3, \ldots$$

Vi noterar att

$$a_k = \begin{cases} 0 \quad , & k = 2n+1, \; n = 1, 2, \ldots \\ \frac{4}{\pi} \frac{1}{1-k^2} \quad , & k = 2n \;, \quad n = 1, 2, \ldots \end{cases}$$

$\Rightarrow$ Fourierserien för $f(x) = |\sin x|$ ges av

$$f(x) \sim \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos kx = \frac{2}{\pi} + \frac{2}{\pi} \sum_{k=2}^{\infty} \frac{1+(-1)^k}{1-k^2} \cos kx$$

$$= \frac{2}{\pi} + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{1-4n^2} \cos(2nx)$$

Vidare gäller att $f(x) = |\sin x|$ styckevis deriverbar på $[-\pi, \pi]$, $2\pi$-periodisk samt kontinuerlig $\forall x \in \mathbb{R}$

$$\Rightarrow \quad f(x) = |\sin x| = \frac{2}{\pi} + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2nx)}{1-4n^2} \quad \forall x \in \mathbb{R}$$

Speciellt har vi för $x = 0$:

$$f(0) = \frac{2}{\pi} + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos(0)}{1-4n^2} = \frac{2}{\pi} + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{1-4n^2} = 0$$

$$\Rightarrow \quad \frac{2}{\pi} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{4n^2-1} = 0 \quad \Rightarrow \sum_{n=1}^{\infty} \frac{1}{4n^2-1} = \frac{1}{2}$$

Svar: $f(x) \sim \frac{2}{\pi} + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2nx)}{1-4n^2}$ och

$$\sum_{n=1}^{\infty} \frac{1}{4n^2-1} = \frac{1}{2}$$