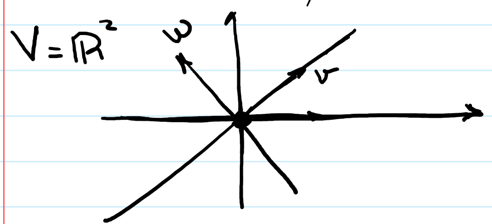
D_AffinM_12_april

den 29 mars 2021 11:53

Def. 1.3 sidan 9

$M \subseteq V$ M affin mängd , $u_0 \in V$. , $U \subseteq V$ underrum.

$$M = u_0 + U = \{ u_0 + u : u \in U \}$$



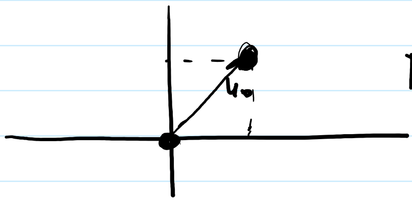
V $U = \{0\}$

$U = \text{Span}\{v\}$, $v \neq 0$

$U = \text{Span}\{v, w\} = V$

$\{v, w\}$ linjärt Oberoende

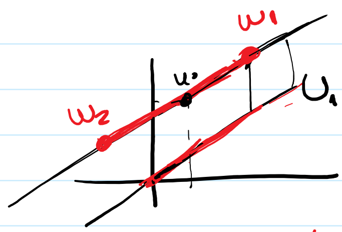
1) $u_0 \neq 0$



$M = u_0 + \{0\}$

$M = \{ u_0 + \vec{0} \}$

2) $u_0 \neq 0$



$U_1 = \text{Span}\{v\} = \{ \lambda v, \ \lambda \in \mathbb{R} \}$

$M = \{ w = u_0 + \lambda v \}$

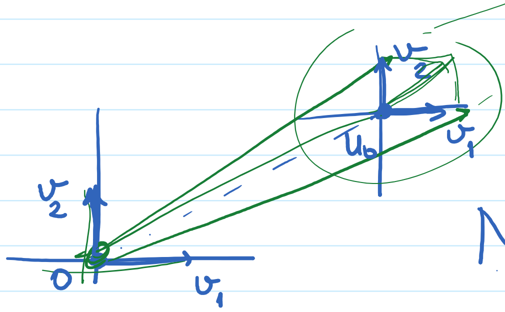
$w_1 = u_0 + \lambda_1 v$

$w_2 = u_0 + \lambda_2 v$

$\Big\} \ w_1 - w_2 = (\cancel{u_0} + \lambda_1 v) - (\cancel{u_0} + \lambda_2 v) =$

$= (\lambda_1 - \lambda_2) v$

3)



$M = \{ \underbrace{w = u_0 + \lambda_1 v_1 + \lambda_2 v_2} \}$

$U_2 = \text{Span}\{u, v\}$

$\dim U_2 = 2$

$w_1 = \cancel{u_0} + \lambda_1 v_1 + \lambda_2 v_2$

$w_2 = \cancel{u_0} + \alpha_1 v_1 + \alpha_2 v_2$

$$w_2 = u_0 + \alpha_1 v_1 + \alpha_2 v_2$$

$$w_2 - w_1 = (\alpha_1 - \lambda_1) v_1 + (\alpha_2 - \lambda_2) v_2$$

Uppgift 9 sidan 44. Vilka delmängden av $\mathbb{R}^4$ är underrum? Vilka är affin mängden?

a) $M = \{ x \in \mathbb{R}^4 : 3x_1 - 2x_2 + x_3 - x_4 = 0 \}$

$$3x_1 - 2x_2 + x_3 - x_4 = 0 \Rightarrow x_1 = -\frac{2}{3}x_2 - \frac{1}{3}x_3 + \frac{1}{3}x_4 \Rightarrow$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -\frac{2}{3}x_2 - \frac{1}{3}x_3 + \frac{1}{3}x_4 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \cancel{x_2}\,\alpha \underbrace{\begin{pmatrix} -2/3 \\ 1 \\ 0 \\ 0 \end{pmatrix}}_{v_1} + \cancel{x_3}\,\beta \underbrace{\begin{pmatrix} -1/3 \\ 0 \\ 1 \\ 0 \end{pmatrix}}_{v_2} + \cancel{x_4}\,\gamma \underbrace{\begin{pmatrix} 1/3 \\ 0 \\ 0 \\ 1 \end{pmatrix}}_{v_3}, \quad \alpha, \beta, \gamma \in \mathbb{R}$$

Då $M = \text{Span}\{v_1, v_2, v_3\} = \left\{ x = (x_1, x_2, x_3, x_4) \,/\, \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \alpha \begin{pmatrix} 2 \\ -3 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ -3 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} 1 \\ 0 \\ 0 \\ 3 \end{pmatrix} \right\}$

M är underrum och $\dim M = 3$; Bas till $M = \left\{ \begin{pmatrix} 2 \\ -3 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -3 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 3 \end{pmatrix} \right\}$

Affin

b) $M_b = \{ x \in \mathbb{R}^4 ; 9x_1 - 2x_2 + x_3 + 4x_4 = 1 \}$

$$9x_1 = 1 + 2x_2 - x_3 - 4x_4$$

$$x_1 = \frac{1}{9} + \frac{2}{9}x_2 - \frac{1}{9}x_3 - \frac{4}{9}x_4$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1/9 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 2/9\, \cancel{x_2}\,\alpha \\ \cancel{x_2}\,\alpha \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -1/9\, \cancel{x_3}\,\beta \\ 0 \\ \cancel{x_3}\,\beta \\ 0 \end{pmatrix} + \begin{pmatrix} -4/9\, \cancel{x_4}\,\gamma \\ 0 \\ 0 \\ \cancel{x_4}\,\gamma \end{pmatrix}$$

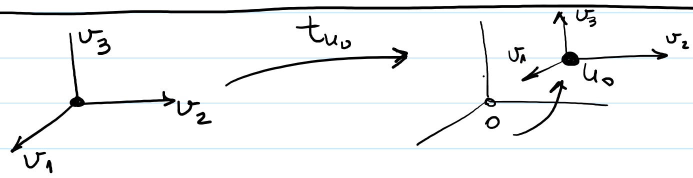
$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \underbrace{\begin{pmatrix} 1/9 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{u_0} + \underbrace{\alpha \begin{pmatrix} 2/9 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} -1/9 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} -4/9 \\ 0 \\ 0 \\ 1 \end{pmatrix}}_{\underbrace{\alpha v_1 + \beta v_2 + \gamma v_3}}$$

$$U = \text{span}\{v_1, v_2, v_3\} =$$

$= \{ \alpha v_1 + \beta v_2 + \gamma v_3 : \alpha, \beta, \gamma \in \mathbb{R} \}$

Då M_b är affin mängd, eftersom M_b kan skrivas som

$$M_b = u_0 + U = \begin{pmatrix} 1/9 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \underbrace{\alpha v_1 + \beta v_2 + \gamma v_3}_{\in U}.$$



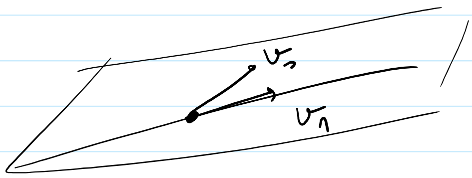
C) $M_c = \{ x \in \mathbb{R}^4 : \underbrace{x = s \overbrace{(2,3,4,5)}^{v_1} + t \overbrace{(6,7,8,9)}^{v_2}} : s, t \in \mathbb{R} \}$

$M_c =$ Underrum

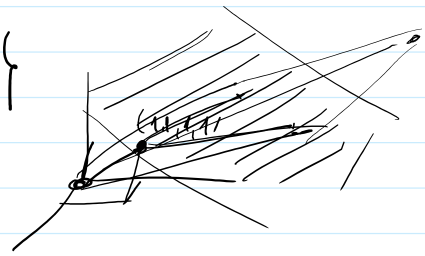
$$\{ x = \cancel{0} + s v_1 + t v_2 \} = \underbrace{\text{Span}\{v_1, v_2\}} \equiv \text{Underrum}.$$

$0 v_1 + s v_1 + t v_2$

$(0+s) v_1 + t v_2$



$\tilde{C}$: $M_{\tilde{c}} = \{ x = (1,1,1,1) + \alpha (2,3,4,5) + t (6,7,8,9) \}$



$y = (1,1,1,1) + \alpha_0 (2,3,4,5) + t_0 (6,7,8,9)$

$z = (1,1,1,1) + \alpha_1 (2,3,4,5) + t_1 (6,7,8,9)$

Då är M_c affin mängd

$y + z = \underbrace{(2,2,2,2)} + (\alpha_0 + \alpha_1) v_1 + (t_0 + t_1) v_2$

OBS. $M = u_0 + U$ affin mängd.

i) $u, v \in M \Rightarrow u + v \notin M$.

ii) $u, v \in M \Rightarrow u - v \in U$

$u = u_0 + \alpha_1 v_1 + \dots \alpha_n v_n$

$v = u_0 + \beta_1 v_1 + \dots + \beta_n v_n$

$u + v = \underbrace{(u_0 + u_0)}_{2u_0 \neq u_0} + (\alpha_1 + \beta_1) v_1 + \dots + (\alpha_n + \beta_n) v_n$

$\therefore u + v \notin M$

$u - v = 0 + \underbrace{(\alpha_1 - \beta_1) v_1 + \dots + (\alpha_n - \beta_n) v_n}_{\in U \text{ underrum.}}$

e) $\{ x \in \mathbb{R}^4 : x = \underbrace{(1,2,0,1)} + t \overbrace{(0,1,2,2)}^{v}, t \in \mathbb{R} \}$

$x = u_0 + \overbrace{t v_1}, t \in \mathbb{R}$

$x = u_0 + U$ $U = \text{Span}\{v\}$.

$$\boxed{x = \underbrace{u_0}_{\text{bestämd}} + U}$$

$U = \text{span}\{v_1\} = \{t v_1,\ t \in \mathbb{R}\}$

$x_1 = \overset{u_0}{(1,2,0,1)} + t_1 v$

$x_2 = \overset{u_0}{(1,2,0,1)} + t_2 v$

$\Longrightarrow M = \{x = u_0 + t v\}$ är affin mängd men inte underrum.

$(x_1+x_2) = \underbrace{(2,4,0,2)}_{\neq u_0} + (t_2+t_1) v$

f) $M_f = \{ x \in \mathbb{R}^4 : x_1 \cdot x_3 = 0 \}$

$v = (x_1, x_2, x_3, x_4)$ $\boxed{x_1 \cdot x_3 = 0}$

$x_1 \cdot x_3 = 0 \Rightarrow x_1 = 0$ eller $x_3 = 0$

$M_f = \underbrace{\{(0, x_2, x_3, x_4)\}}_{U_1 \text{ underrum}} \cup \underbrace{\{(x_1, x_2, 0, x_4)\}}_{U_2 \text{ underrum}}$

$u_1 = (0,1,1,1) \in M$

$u_2 = (1,1,0,1) \in M$

$u_1 + u_2 = (0,1,1,1) + (1,1,0,1) = (1,2,1,2)$

$y_1\ y_2\ y_3\ y_4$

$y_1 \cdot y_3 = 1 \cdot 1 = 1 \neq 0$

Då är M_f ej underrum

OM M var en affin mängd

- M alla element i M måste skrivas som $x = u_0 + U$,
- $x, y \in M \Rightarrow y - x \in U$

$M = \{x \in \mathbb{R}^4 : x_1 \cdot x_3 = 0\} \Rightarrow M = U_1 \cup U_2 = \{x \in \mathbb{R}^4 : x_1 = 0\} \cup \{x \in \mathbb{R}^4 : x_3 = 0\}$

$M = \text{Span}\{(0, x, y, z)\} \cup \text{Span}\{(a, b, 0, c)\}$

1a $x = (0,5,5,5) \in M$
$y = (7,7,0,7) \in M$

$(y-x) = (7,7,0,7) - (0,5,5,5) = (7,2,-5,2) \notin M$

$(y-x) \notin U_1 = \{ (0,x,y,z),\ x,y,z \in \mathbb{R}\}$

$(y-x) \notin U_2 = \{ (a,b,0,c)\ ;\ a,b,c \in \mathbb{R}\}$

Då är M ej affen mängd.

(ej underrum heller

g) $\boxed{x_1^2 + x_2^2 = 0} \Longrightarrow \boxed{x_1 = 0,\ x_2 = 0}$

om $x_1 \neq 0 \Rightarrow x_1^2 + x_2^2 > 0$ oberoende av x_2

om $x_2 \neq 0 \Rightarrow x_1^2 + x_2^2 > 0$ oberoende av x_1

$$M_g = \{ x \in \mathbb{R}^4 : x_1^2 + x_2^2 = 0 \} = \{ x \in \mathbb{R}^4,\ x_1 = x_2 = 0 \}$$

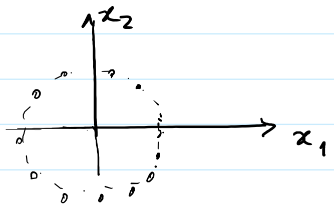
$$= \{ x = (0,0,\alpha_3,\beta_4) \} \equiv \underline{\text{Underrum}}$$

$$\{ x = \overset{x_3}{\alpha}\overbrace{(0\,0\,1\,0)}^{v_1} + \overset{x_4}{\beta}\overbrace{(0\,0\,0\,1)}^{v_2} : \alpha, \beta \in \mathbb{R} \}$$

Span $\{ \underbrace{(0\,0\,1\,0)}_{v_1}, \underbrace{(0\,0\,0\,1)}_{v_2} \}$

M_g är underrum $\dim M_g = 2$ Bas $M_g = \{ (0,0,1,0), (0,0,0,1) \}$

h) $M_h = \{ x \in \mathbb{R}^4 : x_1^2 + x_2^2 = 1 \}$ inte underrum.



$x_1 = 0$
$x_2 = 1$

$\{ x = (x_1, x_2, x_3, x_4) \}$

$$\left.\begin{aligned} p &= (0, 1, x_3, x_4) \\ q &= (1, 0, y_3, y_4) \end{aligned}\right\} p+q = (\underbrace{1, 1}, \alpha, \beta)$$

$$1^2 + 1^2 = 2 \neq 1$$

$M_h = \{ x = \underset{\uparrow}{u_0} + \alpha v_1 + \beta v_2 \dots \}$??? ...

$x = (\cos\pi/4, \sin\pi/4, x_3, x_4)$ $\quad x \in M_h \quad \cos^2\pi/4 + \sin^2\pi/4 = 1$

$y = (\cos\pi/3, \sin\pi/3, y_3, y_4)$ $\quad y \in M_h \quad \cos^2\pi/3 + \sin^2\pi/3 = 1$

$(x+y) = \left(\cos\pi/4 + \cos\pi/3, \sin\pi/4 + \sin\pi/3, (x_3+y_3), (x_4+y_4)\right) \overset{?}{\in} M_h$?

$(\cos\pi/4 + \cos\pi/3)^2 + (\sin\pi/4 + \sin\pi/3)^2 =$

$\cos^2\pi/4 + 2\cos\pi/4\cos\pi/3 + \cos^2\pi/3 + \sin^2\pi/4 + 2\sin\pi/4 \cdot \sin\pi/3 + \sin^2\pi/3 =$

$1 \qquad\qquad 1$

$= 2 + 2(\cos\pi/4\cos\pi/3 + \sin\pi/4\sin\pi/3) \neq 1.$

Då är M_h ej underrum.

OM M_h var en affin mängd, $M = u_0 + U$, då.

$v \in M_h$, $w \in M_h \Rightarrow v - w \in U$

$v = (1, 0, x_3, x_4) \in M$ eftersom $1^2 + 0^2 = 1$

$w = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, y_3, y_4\right) \in M$ eftersom $\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1$

$v - w = \left(1 - \frac{1}{\sqrt{2}}, 0 - \frac{1}{\sqrt{2}}, x_3 - y_3, x_4 - y_4\right) \Rightarrow \left(1 - \frac{1}{\sqrt{2}}\right)^2 + \left(0 - \frac{1}{\sqrt{2}}\right)^2 =$

$$\left(1 - \frac{2}{\sqrt{2}} + \frac{1}{2}\right) + \frac{1}{2} = 2 - \frac{2}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}$$

$$2 - \sqrt{2} \neq 1$$

Då är M ej affinmängd.