L8_V4_22april

den 22 april 2021 09:10

Differensekvation av 2: grad.

$y_{n+2} + \alpha y_{n+1} + \beta y_n = 0$ Homog. Ekvation.

$y_{n+2} + \alpha y_{n+1} + \beta y_n = f(n)$ Inhomong

1° $y_{n+2} + \alpha y_{n+1} + \beta y_n = 0$

Ansats. $\boxed{y_n = r^n}$ $r = ?$

$$\boxed{r^{n+2} + \alpha r^{n+1} + \beta r^n = 0}$$

$r^n(r^2 + \alpha r + \beta) = 0 \implies \underbrace{r^n = 0}$ eller $\underbrace{(r^2 + \alpha r + \beta) = 0}$

$r = 0$ är trivial lösning.

så väljar vi $r \neq 0$

och $\boxed{r^2 + \alpha r + \beta = 0}$

I) $\underbrace{y_{n+1} + \alpha y_n}_{} + \beta y_{n-1} = 0$ 2^{nd} grad differensekvation:

2 steg ∴

Ansats: $\boxed{y_n = r^n}$

$r^{n+1} + \alpha r^n + \beta r^{n-1} = 0$

$r^{n-1}(\underbrace{r^2 + \alpha r + \beta}) = 0$

Karakteristika ekva.

ii) $y_n + \alpha y_{n-1} + \beta y_{n-2} = 0$

$$\boxed{y_n = r^n}$$

$$r^n + \alpha r^{n-1} + \beta r^{n-2} = 0$$

$$r^{n-2}\underbrace{(r^2 + \alpha r + \beta)} = 0 \qquad \boxed{r \neq 0}$$

Karakteristiska ekvation.

Sidan 46 Fibonaccis Talföljd.

$\{F_n\}_{n=0}^{\infty}$ Fibonacci $\begin{cases} F_{n+2} = F_{n+1} + F_n \\ \boxed{F(0) = 0 \quad F(1) = 1} \end{cases} \leftarrow$

$$(F_n)_{n=0}^{\infty} = (0, 1, 1, 2, 3, 5, 8, \ldots) \qquad F_n \in \mathbb{N}.$$

$$\boxed{F_{n+2} - F_{n+1} - F_n = 0}$$

$$ax^2 + bx + c = 0$$

$$r_{1,2} = \frac{-b \pm \sqrt{b^2 - 4.ac}}{2.(a)}$$

$$F_n = r^n \Rightarrow r^2 - r - 1 = 0$$

$$r_{1,2} = \frac{+1 \pm \sqrt{1 - 4.1.(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$r_1 = \frac{1+\sqrt{5}}{2} \qquad r_2 = \frac{1-\sqrt{5}}{2}$$

$r_1 \notin \mathbb{N}$ $\quad r_1 \in \mathbb{R}$

$r_2 \notin \mathbb{N}$ $\quad r_2 \in \mathbb{R}$

$$F_n = A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n$$

$\begin{cases} F(0) = 0 \\ F(1) = 1 \end{cases} \longrightarrow \begin{cases} A.1 + B.1 = 0 \Rightarrow \boxed{A = -B} \\ A\left(\frac{1+\sqrt{5}}{2}\right) + B\left(\frac{1-\sqrt{5}}{2}\right) = 1 \end{cases}$

$$B\left(-\frac{1}{2} - \frac{\sqrt{5}}{2}\right) + B\left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right) = 1$$

$$B\left[-\cancel{\tfrac{1}{2}}+\cancel{\tfrac{1}{2}}-\frac{\sqrt5}{2}-\frac{\sqrt5}{2}\right]=1 \Rightarrow -\frac{\cancel{2}\sqrt5}{\cancel{2}}B=1$$

$$B=-\frac{1}{\sqrt5} \qquad A=+\frac{1}{\sqrt5} \qquad \frac{1}{\sqrt5}=\frac{\sqrt5}{5}$$

$$\boxed{F_n=\frac{\sqrt5}{5}\left(\frac{1+\sqrt5}{2}\right)^n-\frac{\sqrt5}{5}\left(\frac{1-\sqrt5}{2}\right)^n}$$

Obs: $F_n \in \mathbb{N}$ även om $r_1 \notin \mathbb{N}$
$r_2 \notin \mathbb{N}$

$$\lim_{n\to\infty} F_n = \lim_{n\to\infty} \frac{\sqrt5}{5}\left(\frac{1+\sqrt5}{2}\right)^n - \cancel{\frac{\sqrt5}{5}\left(\frac{1-\sqrt5}{2}\right)^n}\to 0 \qquad \left|\left(\frac{1-\sqrt5}{2}\right)\right|<1$$

$$F_n=\frac{1}{\sqrt5}\frac{(1+\sqrt5)^n}{2^n}-\frac{1}{\sqrt5}\frac{(1-\sqrt5)^n}{2^n}=\frac{1}{\sqrt5}\frac{1}{2^n}\left((1+\sqrt5)^n-(1-\sqrt5)^n\right)$$

$$\lim_{n\to\infty}\frac{F_{n-1}}{F_n}=?$$

$$\frac{F_{n-1}}{F_n}=\frac{\cancel{\frac{1}{\sqrt5}\frac{1}{2^{n-1}}}\left((1+\sqrt5)^{n-1}-(1-\sqrt5)^{n-1}\right)}{\cancel{\frac{1}{\sqrt5}\frac{1}{2^n}}\left((1+\sqrt5)^n-(1-\sqrt5)^n\right)}=$$

$$2\,\frac{\cancel{(1+\sqrt5)^{n-1}}\left(1-\cancel{\frac{(1-\sqrt5)^{n-1}}{(1+\sqrt5)^{n-1}}}\right)}{(1+\sqrt5)^{\cancel{n}}\left(1-\cancel{\frac{(1-\sqrt5)^n}{(1+\sqrt5)^n}}\right)}$$

$(1+\sqrt{5})$

$$\boxed{\lim_{n\to\infty} \frac{F_{n-1}}{F_n} = 2/(1+\sqrt{5})}$$

Obs: Om Serier

$$\underbrace{\sum_{k=1}^{\infty} a_k = \text{Series}}$$

$$\boxed{s_n = \sum_{k=1}^{n} a_k} \quad \text{delsumman}$$

talföljd

$S = \sum_{k=1}^{\infty} a_k$ konvergerar om $\boxed{\lim_{n\to\infty} s_n = \alpha}$

$$\boxed{S = \alpha}$$

$$\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$$

$$s_n = \sum_{k=1}^{n} \frac{1}{k(k+1)}$$

formel som är beroende på n

$$\frac{1}{k(k+1)} \overset{*}{=} \underbrace{\frac{1}{k} - \frac{1}{(k+1)}} = \frac{\cancel{k}+1-\cancel{k}}{k(k+1)} = \frac{1}{k(k+1)}$$

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \underbrace{\sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{k+1}\right)}$$

$$\left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) \ldots +$$

$$+ \left(\cancel{\frac{1}{n-1}} - \cancel{\frac{1}{n}}\right) + \left(\cancel{\frac{1}{n}} - \frac{1}{n+1}\right)$$

$$\left(1 - \frac{1}{n+1}\right)$$

lim

$n+1$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(1 - \frac{1}{n+1}\right) = \underline{\underline{1}}$$