MVE545 20511 Dugga

1) a) $\int \sqrt{1-x}\,dx = \begin{bmatrix} \text{direkt, } t = 1-x \\ \text{eller sätt } \frac{dt}{dx} = -1 \; dx \end{bmatrix} = \int \sqrt{t}\,(-dt) =$

$= -\int \sqrt{t}\,dt = -\int t^{1/2}\,dt = -\frac{2}{3} t^{3/2} + C = \boxed{-\frac{2}{3}(1-x)^{3/2} + C}$

b) $\int x\sqrt{1-x^2}\,dx = \begin{bmatrix} \text{inre derivata} \\ t = 1-x^2 \end{bmatrix} = \boxed{-\frac{1}{3}(1-x^2)^{3/2} + C}$

c) $\int \ln x\,dx = [\,PI\,] = x\ln x - \int x \frac{1}{x}\,dx = \boxed{x\ln x - x + C}$

d) $\int \frac{1}{\sqrt{x^2+4}}\,dx = [\,\text{"Listan"}\,] = \boxed{\ln|x + \sqrt{4+x^2}| + C}$

e) $\int \frac{1}{\sqrt{4-x^2}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{1-(\frac{x}{2})^2}}\,dx = \left[t = \frac{x}{2}\right] = \int \frac{1}{\sqrt{1-t^2}}\,dt = [\,\text{"Listan"}\,] =$

$= \arcsin(t) + C = \boxed{\arcsin\left(\frac{x}{2}\right) + C}$

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2) $\begin{array}{r|l} x^2-1 & \overline{x^3 - x + 1} \quad x \\ & -(x^3 - x) \\ & \quad 1 \end{array}$

$\Rightarrow \frac{x^3-x+1}{x^2-1} = \frac{x(x^2-1)+1}{x^2-1} = x + \frac{1}{x^2-1} = x + \frac{1/2}{x-1} - \frac{1/2}{x+1}$

$\therefore \int \frac{x^3-x+1}{x^2-1}\,dx = \int x + \frac{1/2}{x-1} - \frac{1/2}{x+1}\,dx = \frac{x^2}{2} + \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C =$

$= \boxed{\frac{x^2}{2} + \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C,}$ $C \in \mathbb{R}$

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3) a) $y' = \frac{1}{x}y + x^2 \Leftrightarrow y' + (-\frac{1}{x})y = x^2$ linjär IF: $e^{\int -\frac{1}{x}dx} = e^{-\ln|x|} = 1/|x|$

$\therefore \frac{d}{dx}\left(\frac{1}{|x|}y\right) = \frac{1}{|x|}x^2 \Leftrightarrow \frac{d}{dx}\left(\frac{1}{x}y\right) = x \Leftrightarrow \frac{1}{x}y = \int \frac{d}{dx}\left(\frac{1}{x}y\right)dx = \int x\,dx = \frac{x^2}{2} + C$

där $C \in \mathbb{R} \Rightarrow y = x\left(\frac{x^2}{2} + C\right) = \boxed{\frac{x^3}{2} + Cx}$, $C \in \mathbb{R}$

b) $y' = xy^2 + x = x(y^2+1) \Leftrightarrow \frac{1}{y^2+1}\frac{dy}{\cancel{dx}} = x\,dx \Leftrightarrow \int \frac{1}{y^2+1}\,dy = \int x\,dx =$

$= \frac{x^2}{2} + C \Leftrightarrow \arctan(y) = \frac{x^2}{2} + C \Leftrightarrow \boxed{y = \tan\left(\frac{x^2}{2} + C\right),}$ $C \in \mathbb{R}$

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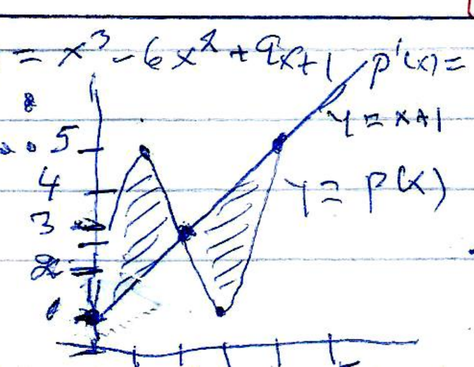
4) $y = x+1$, $y = p(x) = x^3 - 6x^2 + 9x + 1$, $p'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 0$

$\Rightarrow x = 1$ o 3



$A = \int_0^2 x^3 - 6x^2 + 8x\,dx + \int_2^4 -x^3 + 6x^2 - 8x\,dx = \ldots \boxed{= 8}$

$A = \int_0^4 |p(x) - (x+1)|\,dx =$

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