MVE575 Dugga 2018

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1) a) $\int \frac{1}{4x^2+1}\,dx = \int \frac{1}{(2x)^2+1}\,dx = \left[\begin{array}{l} t = 2x \\ \cancel{\frac{dt}{dx}} dt = 2\,dx \end{array}\right] =$

$= \int \frac{1}{t^2+1} \cdot \frac{1}{2}\,dt = \frac{1}{2}\int \frac{1}{1+t^2}\,dt = \frac{1}{2}\arctan t + C =$

$= \boxed{\frac{1}{2}\arctan(2x) + C}$

b) $\int \tan x\,dx = \int \frac{1}{\cos x}\sin x\,dx = \left[\begin{array}{l} t = \cos x \\ \cancel{\frac{dt}{dx}} dt = -\sin x\,dx \end{array}\right] =$

$= -\int \frac{1}{t}\,dt = -\ln|t| + C = \boxed{-\ln|\cos x| + C}$

c) $\int x\cos x\,dx = [PI] = x\sin x - \int 1\cdot\sin x\,dx = \boxed{x\sin x + \cos x + C}$

d) $\int \frac{x}{x^2+1}\,dx = \left[\begin{array}{l} t = x^2+1 \\ \cancel{\frac{dt}{dx}} dt = 2x\,dx \end{array}\right] = \frac{1}{2}\int \frac{1}{t}\,dt = \boxed{\frac{1}{2}\ln|x^2+1| + C}$

e) $\int \ln x\,dx = \int 1\cdot\ln x\,dx = [PI] = x\ln|x| - \int x\frac{1}{x}\,dx = \boxed{x\ln|x| - x + C}$

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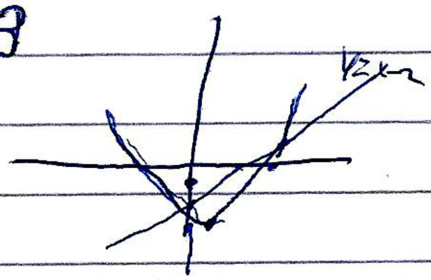
2) $\begin{cases} y = x-2 \\ y = x^2-2x-2 = (x-1)^2-3 \end{cases} \Rightarrow x = 0,\ x = 3$



$A = \int_0^3 x-2-(x^2-2x-2)\,dx =$

$= \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3 = \frac{3^3}{2} - \frac{3^3}{3} - 0 = 3^3\left(\frac{1}{2}-\frac{1}{3}\right) = \frac{3^3}{6} = \boxed{\frac{9}{2}}$

$\int_0^{\pi/4} \tan x\,dx = \left[-\ln|\cos x|\right]_0^{\pi/4} = -\ln\frac{1}{\sqrt{2}} - (-\ln 1)$

$= \boxed{\frac{1}{2}\ln 2}$

MVE545 Dugga Lösn.

3) $I = \int_{-\pi/2}^{\pi/2} \frac{\sin(3x)}{4x^2+1}\,dx$ Låt $f(x) = \frac{\sin(3x)}{4x^2+1}$. Då gäller $f(-x) = -f(x)$ så

f udda $\Rightarrow \boxed{I = 0}$.

4) $\int \frac{x^3-x^2+3}{x^2-2x+2}\,dx = \int x+1+\frac{1}{(x-1)^2+1}\,dx =$

$= \frac{x^2}{2}+x+\int \frac{1}{(x-1)^2+1}\,dx = [t = x-1] =$

$= \frac{x^2}{2}+x+\arctan t + C = \boxed{\frac{x^2}{2}+x+\arctan(x-1)+C}$