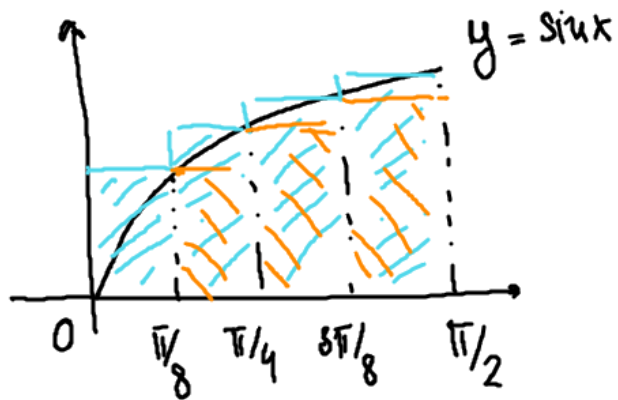


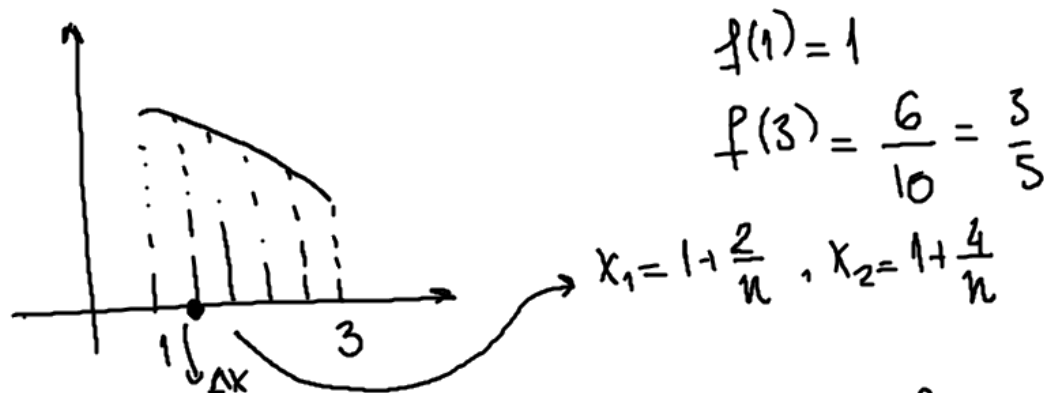
Exempel 1: $f(x) = \sin x$, $[0, \frac{\pi}{2}]$



$$\begin{aligned} R_4 &= \frac{\pi}{8} \cdot \sin\left(\frac{\pi}{8}\right) + \frac{\pi}{8} \cdot \sin\left(\frac{\pi}{4}\right) + \frac{\pi}{8} \cdot \sin\left(\frac{3\pi}{8}\right) + \frac{\pi}{8} \sin\left(\frac{\pi}{2}\right) \\ &= \frac{\pi}{8} \left(\sin\left(\frac{\pi}{8}\right) + \frac{\sqrt{2}}{2} + \sin\left(\frac{3\pi}{8}\right) + 1 \right) \end{aligned}$$

$$r_4 = \frac{\pi}{8} \cdot \sin\left(\frac{\pi}{8}\right) + \frac{\pi}{8} \sin\left(\frac{\pi}{4}\right) + \frac{\pi}{8} \sin\left(\frac{3\pi}{8}\right)$$

Exempel 2: $f(x) = \frac{2x}{x^2+1}$, $1 \leq x \leq 3$



$$f(1) = 1$$

$$f(3) = \frac{6}{10} = \frac{3}{5}$$

$$x_1 = 1 + \frac{2}{n}, x_2 = 1 + \frac{4}{n}$$

$$A = \lim_{n \rightarrow \infty} (f(x_1)\Delta x_1 + f(x_2)\Delta x_2 + \dots + f(x_n)\Delta x_n)$$

$$[1, 3], \text{ längden } 2 \Rightarrow \frac{2}{n} = \Delta x$$

$$A = \lim_{n \rightarrow \infty} \Delta x (f(x_1) + f(x_2) + \dots + f(x_n)) =$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{2(1 + \frac{2}{n})}{(1 + \frac{2}{n})^2 + 1} + \frac{2(1 + \frac{4}{n})}{(1 + \frac{4}{n})^2 + 1} + \dots + \frac{2 \cdot 3}{3^2 + 1} \right)$$