Modelling project/home 2
for the course ODE and mathematical modelling
MVE162/MMG551 in year 2021.
Modeling nonlinear electrical circuits.

The deadline for supplying reports for the second project is the 27-th of May. Alexey Geynts will check both the writing and the mathematical quality of the second project.

The report for a modeling project/assignment must be written as a small scientific article that anybody without deep knowledge in the subject must be able to read.

All Swedish second year students write reports in Swedish.

Master students and exchange students can write reports in English.

Each of you must write your own individual report (in TEX or Word) including:

Introduction to the subject; analytical work, explanation of theoretical background; numerical results with graphical output; interpretation and conclusions.

You are allowed to work on the project in small groups of 2-3 people. Names of the group members must be specified in your individual report.

Uppload your reports and Matlab codes with clear comments at Canvas.

You are welcome to pose questions in Piazza, or contact the teacher by E-mail.

Introduction.

In the second modelling project we consider a classical electromechanical model in two variants:

$$\begin{cases} \dot{x} = y \\ \dot{y} = -x + y - y^3 \end{cases} ; \qquad \text{and} \qquad \begin{cases} \dot{x} = y \\ \dot{y} = -x - y - y^3 \end{cases}$$

A similar equation was introduced earlier by English physicist, Nobel laureate for Physics, John William Strutt in his work on acoustics.

The goal with the project is to apply the whole scale of your knowledge in nonlinear systems to this model. Objects of interest here are: ω - invariant and ω - limit sets, stability of equilibrium points and their attracting regions, and possible existence of periodic solutions.

Read about the first variant of the equation in §1.1.1, pages 2-4 of the course book.

Analytical problems.

1. Find for both equations equilibrium points and investigate their stability and regions of attraction using linearization or Lyapunov´s stability theory.

2. Find if possible ω - invariant sets and ω - limit sets for solutions to both equations. You can use hints in §1.1.1 or can suggest your own ideas for that.

3. Investigate using hints in §1.1.1 and the theory for systems in plane in §4.7, if these equations have periodic solutions.

Numerical problems

4. Create numerically phase portraits for both equations, by choosing appropriate initial points, solving the systems numerically and drawing enough many representative orbits.

5. Compare and possibly complement your theoretical considerations in problems 1, 2, 3 with conclusions based on numerical results.

Hint. It is convenient to choose initial points of trajectories for the phase portrait interactively. One can use for this purpose the Matlab command ginput, that reads coordinates of the mouse pointer on the screen.