Financial Time Series – Order selection for ARMA processes

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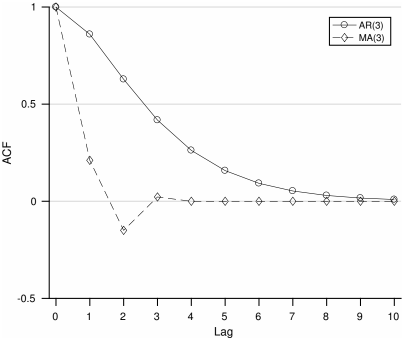
Order selection

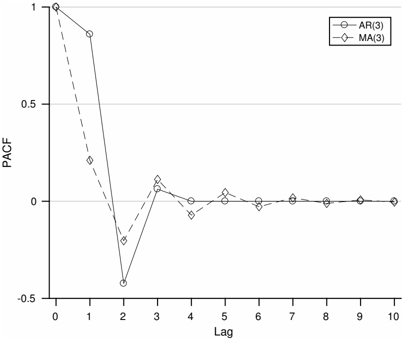
- We know how to estimate $\phi_1, \dots, \phi_p, \theta_1, \dots, \theta_q, \sigma^2$ in

$$X_t - \sum_{j=1}^{p} \phi_j X_{t-j} = Z_t + \sum_{j=1}^{q} \theta_j Z_{t-j}$$

  based on observations $(x_1, \dots, x_n)$ of $(X_1, \dots, X_n)$ but how do we choose p, q?
- Bias $\leftrightarrow$ Variance
- ACF/PACF
- Information criteria (BIC, AICC)



(a) The ACF



(b) The PACF

- For $\mathrm{AR}(p)$: $(\hat{\alpha}(h))_{h \geq p+1} \sim \mathrm{IID}\,\mathcal{N}(0, 1/n)$ approximately
- For $\mathrm{MA}(q)$: $(\hat{\rho}(h))_{h \geq q+1} \sim \mathrm{IID}\,\mathcal{N}(0, 1/n)$
- More systematic: Fit many models, choose the one with a minimum information criterion

Information criterion 1: AICC

Method (AICC criterion)

Choose p, q, ϕ_p, and θ_q to minimize

$$-2\ln L(\phi_p, \theta_q, S(\phi_p, \theta_q)/n) + 2n\frac{p+q+1}{n-p-q-2},$$

where $\phi_p = (\phi_1, \ldots, \phi_p)$ and $\theta_q = (\theta_1, \ldots, \theta_q)$.

- "Corrected" AIC
- Based on the *Kullback–Leibler divergence*
- Efficient: minimizes prediction errors

Information criterion 2: BIC

Method (BIC criterion)

Choose p and q to minimize

$$(n-p-q)\ln\left(n\hat{\sigma}^2/(n-p-q)\right)+n\left(1+\ln\sqrt{2\pi}\right)+(p+q)\ln\left(\left(\sum_{t=1}^{n}X_t^2-n\hat{\sigma}^2\right)/(p+q)\right),$$

where $\hat{\sigma}^2$ denotes the maximum likelihood estimate of the white noise variance.

- Consistent: Will yield the correct model asymptotically

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where $\hat{\sigma}^2$ denotes the maximum likelihood estimate of the white noise variance.

- Consistent: Will yield the correct model asymptotically
- For both criteria, $p+q+1$ is replaced with the *number of nonzero coefficients* if we do not fit a full model

Suggested model building steps for ARMA processes

1. Remove trend and seasonality
2. Identify the order of one or more candidate models
3. Final estimation of candidate model(s)
4. Check goodness of fit and/or out-of-sample prediction error

MLE from last time

- For large n,

$$(\hat{\phi}_1, \ldots, \hat{\phi}_p, \hat{\theta}_1, \ldots, \hat{\theta}_q)' \sim \mathcal{N}((\phi_1, \ldots, \phi_p, \theta_1, \ldots, \theta_q)', 2H^{-1}(\beta)/n)$$

  approximately. Here $H(\beta) = (\delta^2 \ell(\beta)/\delta\beta_i \delta\beta_j)_{i,j=1}^{p+q}$.
- Common alternative: $Z \sim \mathrm{IID}(0, \sigma^2)$ and that Z_t follows a Student t-distribution for all times $t \implies$ *conditional MLE*
- The *rescaled residuals*

$$\hat{R}_t := (X_t - \hat{X}_t)/v_{t-1} = (X_t - \hat{X}_t(\hat{\phi}, \hat{\beta}))/v_{t-1}(\hat{\phi}, \hat{\beta}),$$

  should behave like Z_t/σ
- $\mathbb{E}((\hat{R}_t - Z_t/\sigma)^2) \to 0$