Partial answer sheet

Below are the answers to some of the more computational exercises. If you spot any mistakes, please contact andreas.petersson [at] chalmers . se. The amount of details is not representative for what would be needed from a similar question on the exam.

Below [BD] means the book by Brockwell and Davis and [E] means the PDF "Extra Exercises in Basic Probability for Financial Time Series".

[BD] 1.4: a) Yes. b) Yes. c) No. d) Yes. e) No. f) Yes.

[BD] 1.6: a) 4.6375 b) 0.1257

[E] 1: a) $\frac{1}{2}\sigma^2$ b) a) $\frac{1}{8}\sigma^2$

[BD] 2.3: a) $\gamma(0) = 1.25$, $\gamma(1) = 0.18$, $\gamma(2) = -0.4$, $\gamma(h) = 0$ for $|h| > 2$. b) Same as in a).

[BD] 2.21 Let in this exercise a_i be the coefficient in front of X_i. a) $a_1 = -\frac{\theta^2}{\theta^4+\theta^2+1}$, $a_2 = \frac{\theta(\theta^2+1)}{\theta^4+\theta^2+1}$ b) $a_4 = \frac{\theta(\theta^2+1)}{\theta^4+\theta^2+1}$, $a_5 = -\frac{\theta^2}{\theta^4+\theta^2+1}$ c) Coefficients coincide with answers from parts a and b. d) The MSE for a and b is $\frac{(\theta^2+1)(\theta^4+1)\sigma^2}{\theta^4+\theta^2+1}$ and for c it is $\frac{(\theta^6+1)\sigma^2}{\theta^4+\theta^2+1}$

[BD] 3.1: a) Causal and invertible. b) Not causal but invertible. c) Not invertible but causal. d) Causal and invertible. e) Not causal but invertible.

[BD] 3.3: a) $(-0.2, 0.52, -0.2, 0.2896, -0.15392)$ c) $(0.6, -0.36, 0.216, -0.1296, 0.07776)$ d) $(-1.8, 2.43, -2.916, 3.2805, -3.54294)$

[BD] 3.4: For $h \in \mathbb{Z}$, $\rho(h) = \begin{cases} 0.8^{|h|/2}, \ h \text{ even} \\ 0, \ h \text{ odd} \end{cases}$. For $h \in \mathbb{N}$ $\alpha(0) = 1$, $\alpha(2) = 0.8$ and $\alpha(h) = 0$ otherwise.

[BD] 5.3: a) For $|\phi| < \frac{\sqrt{5}-1}{2}$. b) $\hat{\phi} = 0.509, \hat{\sigma^2} = 2.985$.

[BD] 5.4: a) No. b) $\hat{\mu} = 3.82$, $\hat{\phi_1} = 0.274$, $\hat{\phi_2} = 0.358$ $\hat{\sigma^2} = 0.820$ d) 0.274 ± 0.129 and 0.358 ± 0.129 e) $\hat{\alpha}(1) = \hat{\rho}(1)$, $\hat{\alpha}(2) = \phi_2$ and $\hat{\alpha}(h) = 0$ otherwise.

[BD] 5.11: $\hat{\phi} = \frac{2x_1x_2}{(x_1^2+x_2^2)}, \hat{\sigma^2} = \frac{(x_1^2-x_2^2)^2}{2(x_1^2+x_2^2)}$.

Nonlinear model exercises: 1: a) 13.86 b) 13.55 2: Null hypothesis of a linear ARMA model being adequate is not rejected at the 5% level, with a p-value of 0.0760 (the p-value is found with a suitable statistics software package, while critical values for the test can be found in any statistics textbook).