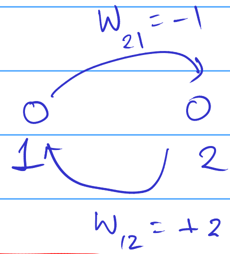
Solutions to exam: October 28, 2019

1. Energy function in a Neural Network.



<u>Given</u> $w_{12} = 2$, $w_{21} = -1$

$$H = -\frac{(w_{12} + w_{21})}{2} S_1 S_2 ,$$

$$\boxed{\begin{aligned} S_1' &= \mathrm{sgn}(w_{12} S_2) \\ S_2' &= \mathrm{sgn}(w_{21} S_1) \end{aligned}}$$

<u>Part (a) 1</u>: Update S_1:

$$\Delta H = H' - H = -\frac{(w_{12} + w_{21})}{2} \left[S_1' S_2 - S_1 S_2 \right]$$

$$= -\frac{(2 - 1)}{2} \left[\mathrm{sgn}(w_{12} S_2) - S_1 \right] S_2$$

$$= -\frac{1}{2} \left[\mathrm{sgn}(2 S_2) - S_1 \right] S_2$$

$$= -\frac{1}{2} \left[S_2 \, \mathrm{sgn}(2 S_2) - S_1 S_2 \right]$$

now, if $S_2 = S_1$, $\Delta H = 0$.

if $S_2 \neq S_1$, $\Delta H < 0$.

Similar argument is used for the S_2 update: one finds

$$\Delta H = -\frac{1}{2} \left[S_1 \, \mathrm{sgn}(-S_1) - S_1 S_2 \right]$$

if $S_1 = S_2$, $\Delta H > 0$ +1

if $S_1 \neq S_2$, $\Delta H = 0$.

(b) $\Delta H = H' - H = -\frac{1}{2} (w_{12} + w_{21}) \left[S_1' S_2' - S_1 S_2 \right]$

$$= -\frac{1}{2} \left[\mathrm{sgn}(2 S_2) \, \mathrm{sgn}(-S_1) - S_1 S_2 \right]$$

Now note that $\mathrm{sgn}(2 S_2) \, \mathrm{sgn}(-S_1) \neq S_1 S_2$ for any $S_1, S_2 = \{\pm 1\}$.

thus $\Delta H \neq 0$.

(c) Follow the same steps as section 2.5 in the book. (no thresholds there). The result for the case with thresholds is given in Equation 4.4 in chapter 4.

2(a) Hamming distance between patterns (μ) & (ν) with bits ± 1 : $\boxed{D_{\mu\nu} = \frac{1}{4}\sum_{i=1}^{N}\left(x_i^{(\mu)} - x_i^{(\nu)}\right)^2}$

Hamming distance counts the number of bits at which 2 patterns differ.

$$D_{\mu\nu} = \frac{1}{4}\sum_{i=1}^{N}\left(x_i^{(\mu)2} + x_i^{(\nu)2} - 2x_i^{(\mu)}x_i^{(\nu)}\right)$$

$$= \frac{1}{2}\sum_{i=1}^{N}\left(1 - x_i^{(\mu)}x_i^{(\nu)}\right)$$

$$= \frac{N}{2} - \frac{1}{2}\sum_{i=1}^{N} x_i^{(\mu)}x_i^{(\nu)}$$

$$\Rightarrow \boxed{\sum_{i=1}^{N} x_i^{(\mu)}x_i^{(\nu)}} = N - 2D_{\mu\nu}$$

$\mu=1,\ \nu=2$: $= 35 - 2\cdot 10 = 15$

$\mu=1,\ \nu=3$: ....

w_{ij} is computed using Hebbs rule for patterns 1 & 2 : $w_{ij} = \frac{1}{N}\sum_{\mu=1}^{2} x_i^{(\mu)}x_j^{(\mu)}$

(b) $$b_i^{(\nu)} = \sum_{j=1}^{N} w_{ij}x_j^{(\nu)} = \frac{1}{N}\sum_{j=1}^{N}\sum_{\mu=1}^{2} x_i^{(\mu)}x_j^{(\mu)}x_j^{(\nu)}$$

$$= \frac{1}{N}\sum_{\mu=1}^{2} x_i^{(\mu)}\boxed{\sum_{j=1}^{N} x_j^{(\mu)}x_j^{(\nu)}}$$

$$= \frac{1}{N}\left[x_i^{(1)}\sum_{j=1}^{N} x_j^{(1)}x_j^{(\nu)} + x_i^{(2)}\sum_{j=1}^{N} x_j^{(2)}x_j^{(\nu)}\right]$$

$\nu=1$

$$b_i^{(1)} = \frac{1}{N}\left[x_i^{(1)}\sum_{j=1}^{N} x_j^{(1)}x_j^{(1)} + x_i^{(2)}\sum_{j=1}^{N} x_j^{(2)}x_j^{(1)}\right]$$

$$= \frac{1}{N}\left[x_i^{(1)}N + x_i^{(2)}\cdot 15\right]$$

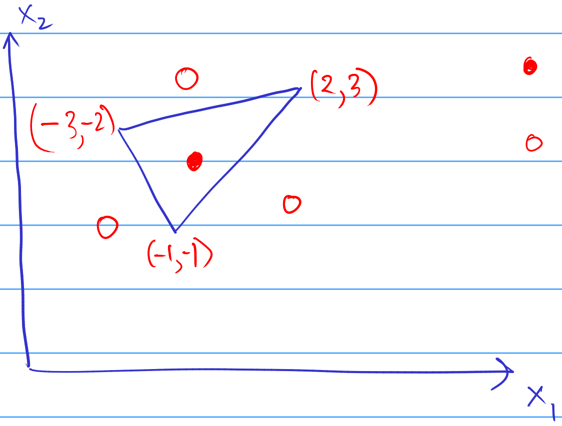
$$\boxed{b_i^{(1)} = x_i^{(1)} + \frac{15}{35} x_i^{(2)} \cdot} \ \ldots$$

c). Use results from (b). Compare with expectation from the book.

$\text{sgn}(b_i^{(1)}) = x_i^{(1)}$ 1, 2, 5 should remain the same, others: not.

3. (a)

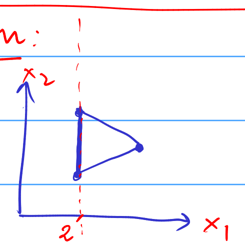
Choose points: filled circle & empty circle can not be separated by a line.



● $t = +1$

○ $t = -1$

Question:



① Write the eq^n of the decision

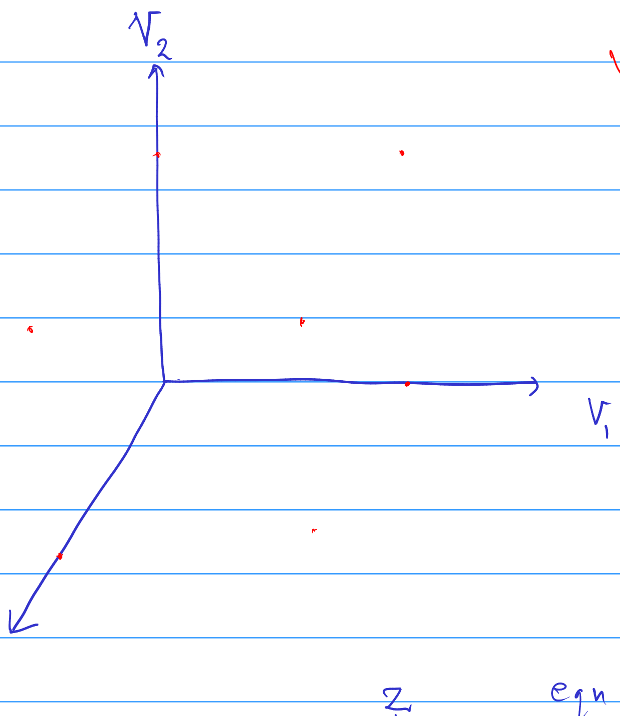
eqn: $x_1 - 2 = 0$

eqn: $x_1 + 0x_2 - 2 = 0$

$w = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\theta = 2$

(b) Same as Problem 5.6 in the book.

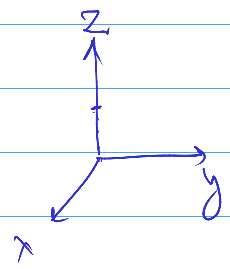
(c)



$v_1 = \text{sgn}(b_1)$

$b_1 = \sum_{j=1}^{2} w_{1j} x_j - \theta_1$

$\boxed{\text{Ex. } 5.7.}$

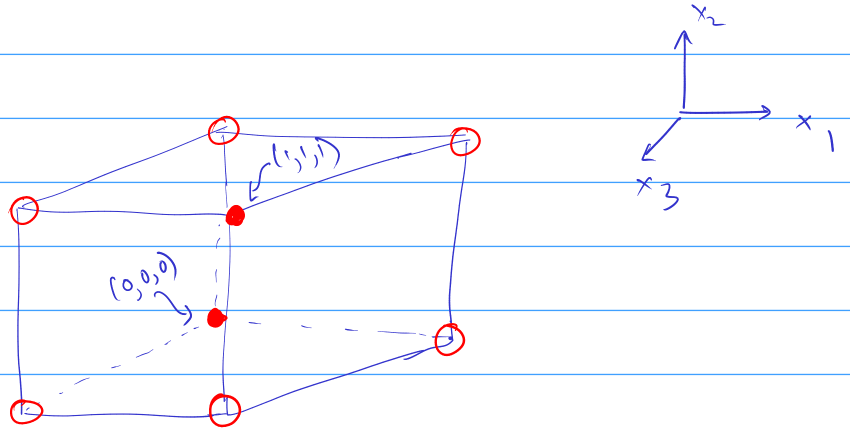


eqn of plane: $ax + by + cz + d = 0$

if $x = 0 = y$

$\underset{(x=0, y=0)}{z} = \frac{-d}{c}$

(a)



(b)

$$g_1(x^{(n)}) = e^{-|x^{(n)} - (0,0,0)|^2}$$

$$g_2(x^{(n)}) = e^{-|x^{(n)} - (1,1,1)|^2}$$

n	x_1	x_2	x_3	t	g_1	g_2
3	0	0	0	+1	1	$e^{-3} \approx 0.05$
8	1	1	1	+1	e^{-3}	1
6	0	0	1	−1	e^{-1}	e^{-2}
1	0	1	0	−1	e^{-1}	e^{-2}
4	1	0	0	−1	e^{-1}	e^{-2}
7	0	1	1	−1	e^{-2}	e^{-1}
2	1	0	1	−1	e^{-2}	e^{-1}
5	1	1	0	−1	e^{-2}	e^{-1}