Exercise: ATE (solution)

Since we know that X is an adjustment set, we can identify the effect of T on Y by conditioning on X. Technically, we also need to assume *consistency*.

By the adjustment formula, we have that

$$\text{ATE} = \mathbb{E}_X\big[\mathbb{E}_Y[Y \mid X, T=1] - \mathbb{E}_Y[Y \mid X, T=1]\big]$$

and

$$\text{CATE(x)} = \mathbb{E}_Y[Y \mid X=x, T=1] - \mathbb{E}_Y[Y \mid X=x, T=1]$$

Since both are functions of $\mu_t(x) = \mathbb{E}_Y[Y \mid X=x, T=t]$, we estimate this function.

By definition, for this example, since $y \in \{0,1\}$

$$\mu_t(x) = \sum_y p(Y=y \mid X=x, T=t)y = p(Y=1 \mid X=x, T=t)$$

From the tables, we get

$$\mu_0(0) = \frac{40}{90+40} = \frac{4}{13}$$

$$\mu_0(1) = \frac{100}{50+100} = \frac{2}{3}$$

$$\mu_1(0) = \frac{10}{10+10} = \frac{1}{2}$$

$$\mu_1(1) = \frac{50}{100+50} = \frac{1}{3}$$

$T = 0$

	$X=0$	$X=1$
$Y=0$	90	50
$Y=1$	40	100

$T = 1$

	$X=0$	$X=1$
$Y=0$	10	100
$Y=1$	10	50

Thus, we have

$$\text{CATE}(0) = \frac{1}{2} - \frac{4}{13} = \frac{5}{26}$$

$$\text{CATE}(1) = \frac{1}{3} - \frac{2}{3} = -\frac{1}{3}$$

For the ATE, we have $\text{ATE} = p(X=0)\text{CATE}(0) + p(X=1)\text{CATE}(1)$. From the table, we have that

$$p(X=0) = \frac{90+40+10+10}{90+40+10+10+50+100+100+50} = \frac{1}{3}$$

and thus $p(X=1) = \frac{2}{3}$.

Hence,

$$\text{ATE} = \frac{1}{3}\cdot\frac{5}{26} - \frac{2}{3}\cdot\frac{1}{3} = -\frac{37}{234} \approx -0.158 \quad .$$

□