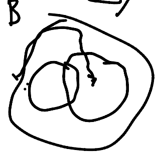
1. E.T. Let $A_1, A_2, \dots A_k$ be open balls in $\mathbb{R}^2$. Then $\exists$ a ~~subset~~ collection of these $B_1, \dots B_\ell$ which (1) disjoint and

(2) $m\left(\bigcup_{i=1}^{\ell} B_i\right) \ge \frac{1}{9} m\left(\bigcup_i A_i\right)$

If B is a ball B^* is the ball with same center as B but 3 times radius

Lemma: A, B balls, $A \cap B \ne \emptyset$, $r_A \le r_B \implies A \subseteq B^*$.



$$\le r_B + 2 r_A \le 3 r_B. \quad \square$$

Let B_1 be the A_i with largest radius. Remove all A_is which intersect B_1. Let B_2 be the largest of the remaining A_is.

Remove A_is which intersect B_2. Choose B_3 largest of the remaining . . . . call them $B_1 \dots B_\ell$. Disj. by construction.

Claim Each A_i is contained in some B_j^* ($\bigcup A_i \subseteq \bigcup B_j^*$)

Pf. case 1: A_i one of the B_j's.

case 2: . . . not one of the B_j's.

So A_i was removed at some pt. Let's say remove 3rd step

$$A_i \cap B_3 \ne \emptyset \implies A_i \subseteq B_3^*$$

if $\operatorname{rad}(A_i) \le \operatorname{rad}(B_3)$

true since O.W. would have chose A_i instead of B_3.

Final step

$$m\left(\bigcup_i A_i\right) \leq m\left(\bigcup_j B_j^*\right) \leq \sum_j m(B_j^*)$$

$$= \sum_j 9\; m(B_j) \underset{\text{disjoint}}{=} 9\; m\left(\bigcup_j B_j\right)$$

---

works in R^n.

$$m\left(\bigcup B_j\right) \geq \frac{1}{3^n}\; m\left(\bigcup A_i\right)$$