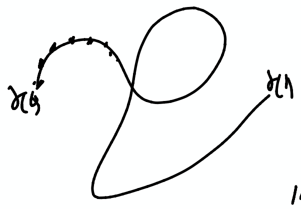
Motivation/review

arc length of paths.

$\gamma : [0,1] \longrightarrow \mathbb{R}^2$

$t \longrightarrow \gamma(t)$



Length of γ.

$0 = t_0 \ t_1 \ t_2 \ \ldots \ t_n = 1$

"$\sum_{i=1}^{n} \|\gamma(t_i) - \gamma(t_{i-1})\|$" should approx well.

$L(\gamma) =$ " $\lim_{\underbrace{\max|t_i - t_{i-1}|}_{\text{mesh}} \to 0 \ (n \to \infty)}$ " "Riemann sums"

$t_i \ \tilde{t} \ t_{i+1}$

$L(\gamma)$: total length moved by a particle

Note if we add 1 pt to our partition, then all terms same except we remove

$\|\gamma(t_i) - \gamma(t_{i+1})\|$ and we add

$\le$ by $\triangle$

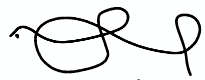
$\|\gamma(t_{i+1}) - \gamma(\tilde{t})\| + \|\gamma(\tilde{t}) - \gamma(t_i)\|$

$\triangle$ refining a partition can only increase $\sum$.

$L(\gamma) = \sup\left(\sum_{i=1}^{n} \|\gamma(t_i) - \gamma(t_{i-1})\| \right.$: as $0 < t_1 < t_2 \cdots < t_n$

Saw a formula in multi dim. calc.

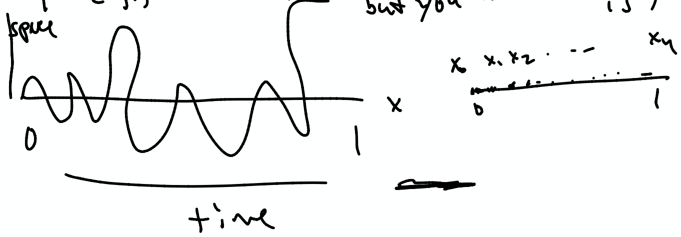
$L(\gamma)$ can be ∞.



$L(\gamma) < \infty \iff \gamma$ is rectifiable

Any dim. Want to do in 1-d. In 1-d, we include the time parameter in our picture.

$f: [0,1] \to \mathbb{R}$ (not necess cont but you can think it is)



$$\sum_{i=1}^{n} |f(x_i) - f(x_{i-1})|$$

~~any~~ adding a point to our partition only can increase the sum

$TV(f) = \sup\{ \sum_{i=1}^{n} |f(x_i) - f(x_{i-1})| : x_1 \ldots x_n \}$

$\parallel$

Length of the path in 1-d

Def: f has finite total variation if $TV(f) < \infty$.

"Formula" $TV(f) = \int_0^1 |f'(x)|\,dx$ if f is cont diff $\iff f'$ cont $\iff f \in C^1$

$TV(f) \approx \sum_{i=0}^{n} |f(x_i) - f(x_{i-1})|$ fine part.

$= \sum_{i=0}^{n} |f'(y_i)| [x_i - x_{i-1}]$ $y_i \in [x_{i-1}, x_i]$

meanvalue thm

$\approx \int_0^1 |f'(y)|\,dy$

since Riemann sum

1. f is monoton $\Rightarrow TV(f) < \infty$ even if f discont

$\sum_{i=1}^{n} |f(x_{i+1}) - f(x_i)|$

$= \sum_{i=1}^{n} f(x_i) - \cancel{f(x_i)}\ f(x_{i-1})$

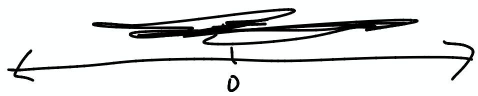
$= f(x_n) - f(x_0)$

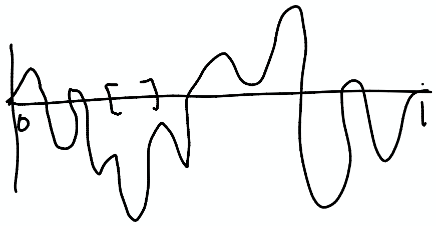
$= f(1) - f(0)$

$\Rightarrow$

$TV(f) = f(1) - f(0)$.

B. M.





B(t)

(no where diff) a.s.

total variation of the path will be inf. on every subinterval a.s.

Bottom Line!

there will be a 1-1 correspondence between function of finite Total variation and signed measures on [0,1]