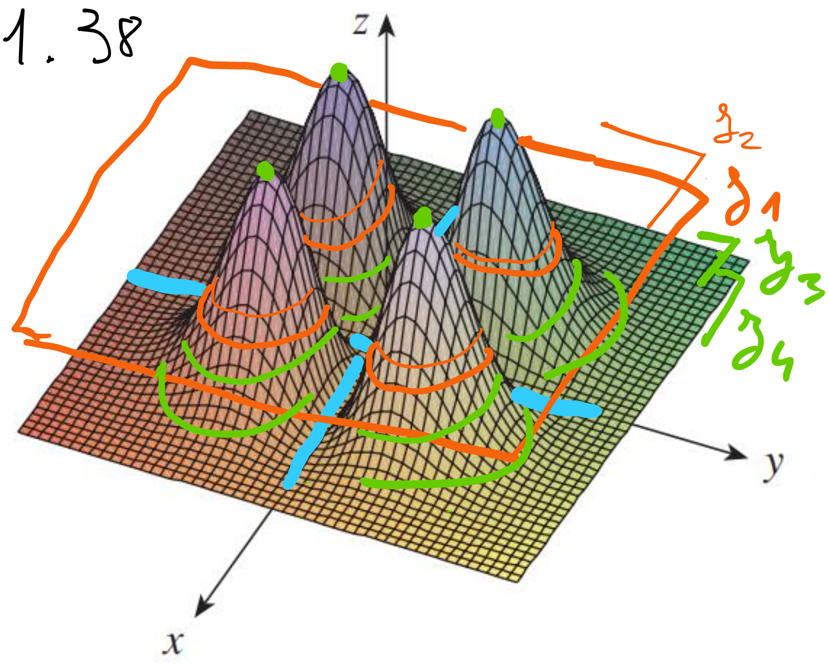
14.1.38

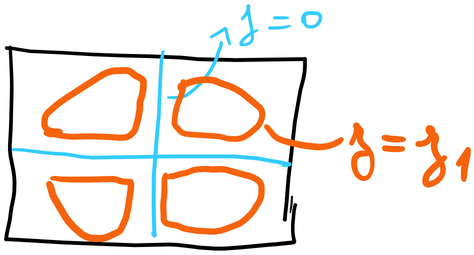


1) x-axis and y-axis stay at 0

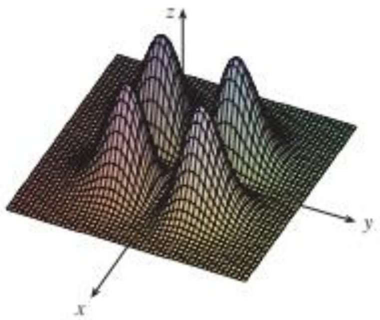
2) there is $m \in \mathbb{R}^+$ such that: $f(x,y) < m$ for all x,y

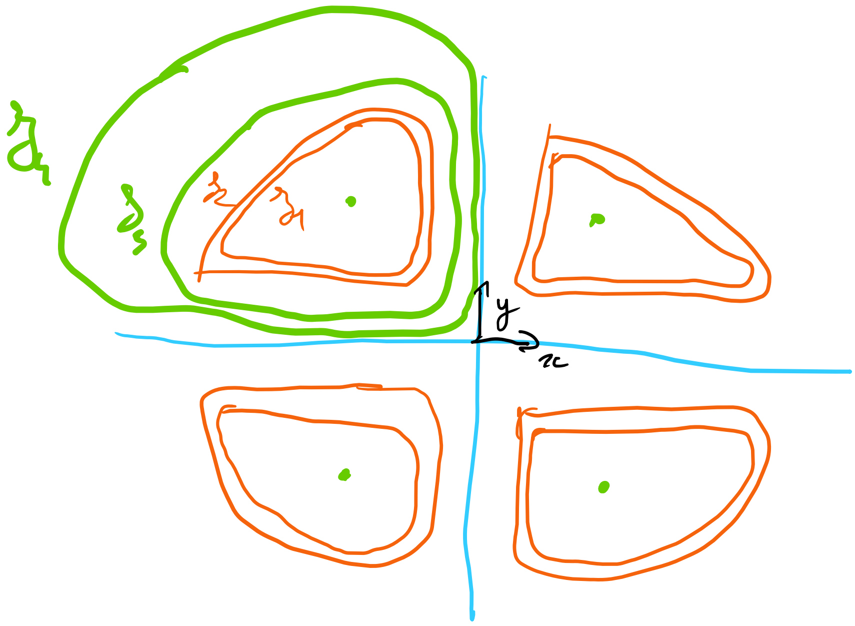
$$z = f(x,y)$$

$$\{(x, y, f(x,y)), (x,y) \in \mathbb{R}^2\}$$

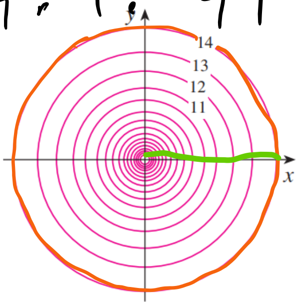


38. Make a rough sketch of a contour map for the function whose graph is shown.

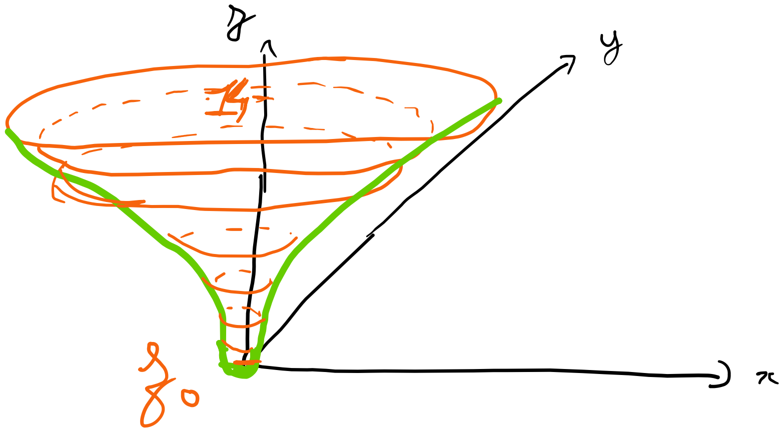


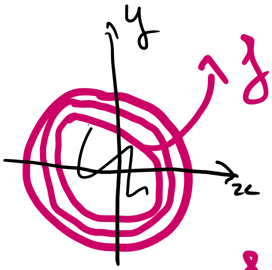


14.1.41



Draw a sketch of the graph from the contour plot

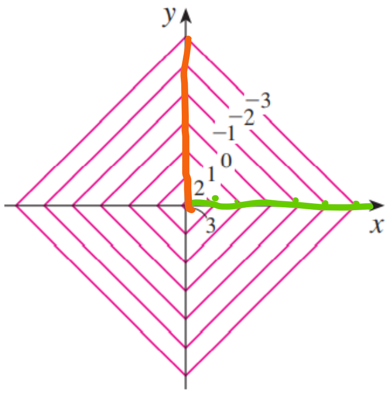




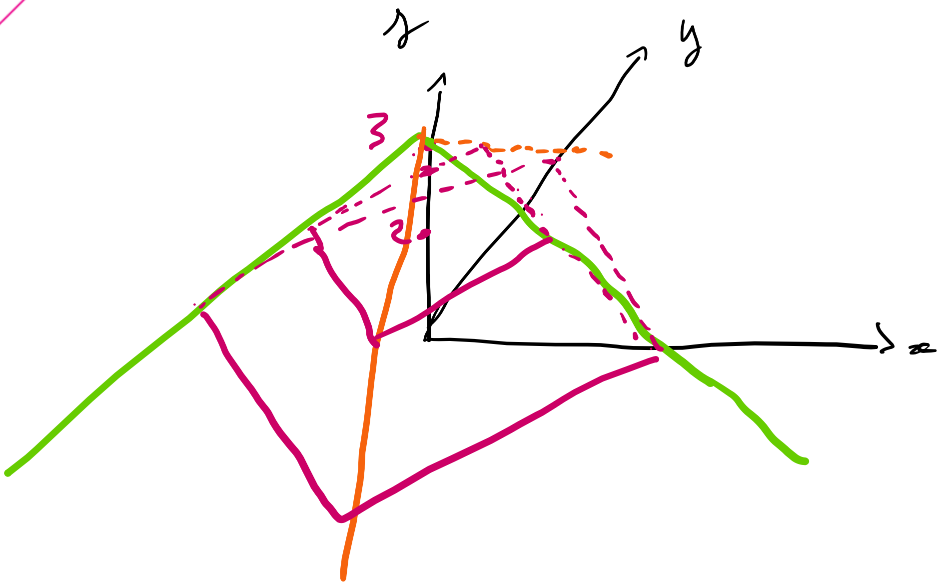
$z = z_0$

$z < z_0$

14.1.44



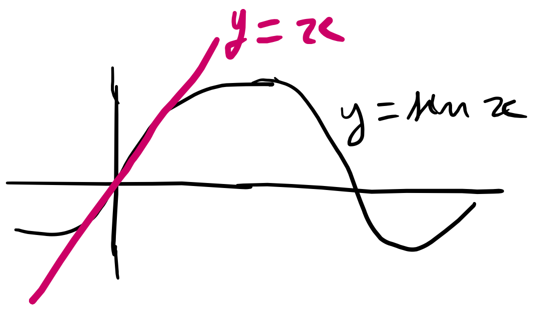
Draw the graph from the contour plot



14.2.10

Compute the limit if it exists of

$$\lim_{(x,y)\to(0,0)} \frac{x^2+\sin^2 y}{2x^2+y^2}$$



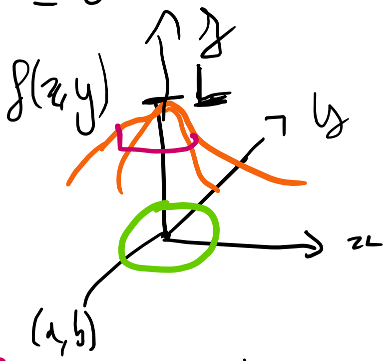
Recall

Let f a function defined on $D\subset\mathbb{R}^2$ and $(a,b)\in D$ such there are points arbitrarily close to (a,b) in D

then the limit of $f(x,y)$ as (x,y) approaches (a,b) is L

if $\boxed{\forall \varepsilon>0\ \exists\delta>0 \text{ s.t } \forall (x,y)\in D \text{ s.t } \sqrt{(x-a)^2+(y-b)^2}\le\delta \text{ then } |f(x,y)-L|\le\varepsilon}$

$$(x-a)^2+(y-b)^2\le\delta^2$$



$\boxed{\text{for } y=0,\ x\neq 0}$ $\quad f(x,0)=\dfrac{x^2}{2x^2}=\dfrac{1}{2}\xrightarrow[x\to 0]{}\dfrac{1}{2}$

$\boxed{\text{for } x=0,\ y\neq 0}$ $\quad f(0,y)=\dfrac{\sin^2 y}{y^2}$

L'Hopital Rule

if we have a quotient $\dfrac{h}{g}$ and $h\to 0$ and $g\to 0$ or $h\to\infty$ and $g\to\infty$

then $\lim \dfrac{h}{g}=\lim\dfrac{h'}{g'}$

here for $h(y) = \sin^2(y)$ and $g(y) = y^2$

$h'(y) = 2\cos(y)\sin(y)$ $\quad g'(y) = 2y$

$$\lim_{y\to 0} \frac{h}{g}(y) = \lim_{y\to 0} \frac{h'}{g'}(y) = \lim_{y\to 0} \frac{2\cos(y)\sin(y)}{2y}$$

L'hopital rule again

$j(y) = 2\cos(y)\sin(y)$ $\quad j'(y) = -2\sin^2(y) + 2\cos^2(y)$

$k(y) = 2y$ $\quad k'(y) = 2$

$$\lim_{y\to 0} \frac{h}{g}(y) = \lim \frac{j'(y)}{k'(y)} = \frac{2}{2} = \underline{1}$$

14.2.16

$$\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^4+y^4}$$

1) $(x^2+y^2)^2 = x^4 + 2x^2y^2 + y^4 \geqslant x^4 + y^4$

for $(x,y) \neq (0,0)$

$$\frac{xy^4}{x^4+y^4} \leqslant \frac{xy^4}{(x^2+y^2)^2} = x \underbrace{\left(\frac{y^2}{x^2+y^2}\right)^2}_{\leqslant 1}$$

2) $x^2+y^2 \geqslant y^2$

We want to prove that $\frac{xy^4}{x^4+y^4} \xrightarrow[(x,y)\to(0,0)]{} 0$

Let $\varepsilon > 0$

$$\left| \frac{xy^4}{x^4+y^4} - \overset{0}{\overset{\shortparallel}{L}} \right| = \left| \frac{xy^4}{x^4+y^4} \right| \le |x|$$

$$\le \sqrt{x^2}$$

$$\le \sqrt{x^2+y^2}$$

$$\le \sqrt{(x-0)^2+(y-0)^2}$$

We need to choose δ such that

$\sqrt{(x-0)^2+(y-0)^2} \le \delta$ then $\left| \frac{xy^4}{x^4+y^4} - 4 \right| \le \varepsilon$

With $\delta = \varepsilon$ then

$$\left| \frac{xy^4}{x^4+y^4} \right| \le \delta = \varepsilon$$

So $\lim\limits_{(x,y)\to(0,0)} \frac{xy^4}{x^4+y^4} = 0$