section 14.2 : 34, 38, 40, 25

section 14.3 : 14.3.

14.2 : 34. Determine the set of points at which the f-n is contin.

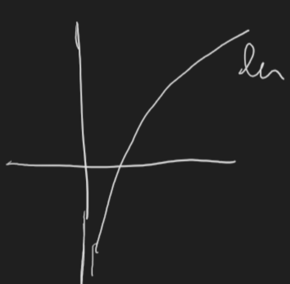
$$G(x,y) = \ln(1+x-y).$$

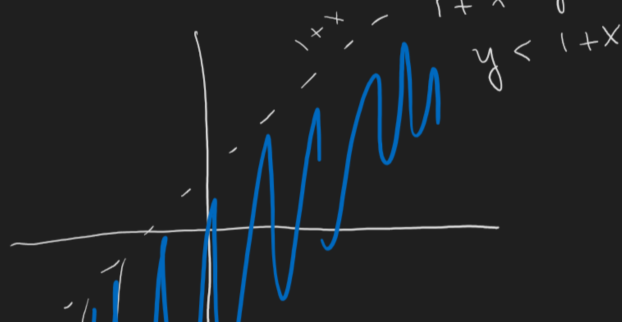
Solution Recall : G is contin. at (a,b) if $\lim_{(x,y)\to(a,b)} G(x,y) = G(a,b)$.

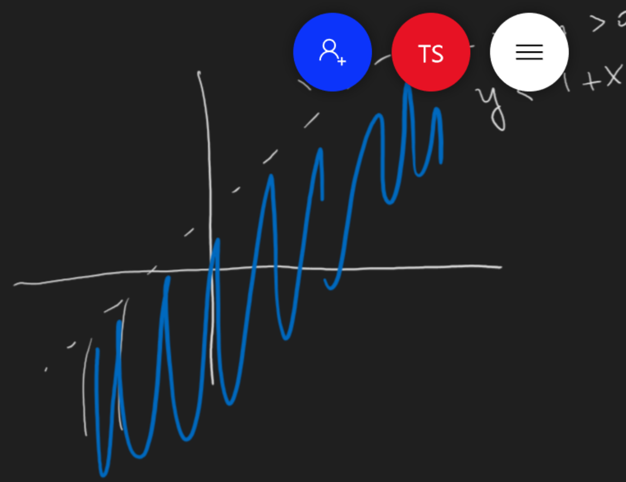
Remark : G can be contin. only at points of its domain.

At first, let us find the domain :

$$D = \{(x,y) \mid 1+x-y > 0\}.$$







G is the composition of 2 f-s which are contin. on their domains (namely of ln and of 1+x-y). Hence by one of Continuity laws G is also contin. on its domain D, which is found above.

Section 4.2. : 38 Determine the set of pts at which the f-n is contin.

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+xy+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

Solution $D = \{(x,y) \mid x^2+xy+y^2 \neq 0,\ (x,y) \neq (0,0)\} \cup$

Solution $D = \{(x,y) \mid x^2+xy+y^2 \neq 0, (x,y)\neq(0,0)\} \cup \{(0,0)\}$

union

| means "and"

At first let us find $(x,y) \neq (0,0)$ s.t.

$x^2+xy+y^2=0$

say $x \neq 0$. Divide by x^2:

$1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2 = 0.$

$D = 1-4 < 0$

— no solutions.

$at^2+bt+c=0$

$D = b^2-4ac$. <0 no solutions.

otherwise $t = \frac{-b \pm \sqrt{D}}{2a}$.

Hence is never 0.

Hence $D = \{(x,y) \mid (x,y)\neq(0,0)\} \cup \{(0,0)\}$

$= \mathbb{R}^2$.

Take at first $(x,y) \neq (0,0)$. For such pt f is given by the f-la $\frac{xy}{x^2+xy+y^2}$, which is the ratio of 2 polynomials, hence is contin. by a Continuity Law.

For investigating continuity at (0,0) we have to use the definition of continuity.

$$\lim_{(x,y)\to(0,0)} f(x,y) \stackrel{?}{=} f(0,0)$$

$$\stackrel{?}{=} 0 \qquad (1)$$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+xy+y^2}$$

Along the path $x=0$: $\frac{xy}{x^2+xy+y^2} = 0 \to 0$

Along the path $y=x$: $\frac{xy}{x^2+xy+y^2} = \frac{x^2}{3x^2} = \frac{1}{3} \neq 0$.

Hence the limit $\nexists$, hence (1) does hold

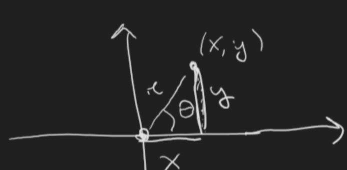
So f is not cont. at (0,0).

Section 4.2: 40 Use polar coordinates to find the limit

$$\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2)$$

θ, r.

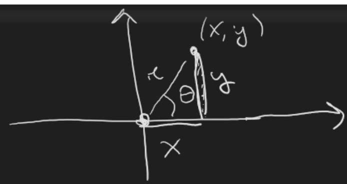
Solution



$$r = \sqrt{x^2+y^2}$$

$\tan\theta = \frac{y}{}$

Solution



$$r = \sqrt{x^2+y^2}$$

$$tg\ \theta = \frac{\ \ }{x}$$

$$x = r\cos\theta$$
$$y = r\sin\theta.$$

In our case: $x^2+y^2 = \boxed{r^2}$.

In polar coordinates our f-n is $r^2 \ln(r^2)$.

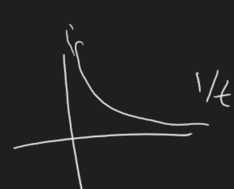
Instead let us denote $t := x^2+y^2$, then we will get even an easier expression

$$\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2) =$$

$$= \lim_{t\to 0} \underbrace{t}_{0}\ \underbrace{\ln t}_{-\infty} =$$

If $(x,y)\to(0,0)$, then $t = x^2+y^2 \to 0$.

We have "$0 \cdot \infty$" case.



$$= \lim_{t\to 0} \frac{\ln t}{\frac{1}{t}} \overset{\text{l'Hospital}}{=} \lim_{t\to 0} \frac{\frac{1}{t}}{-\frac{1}{t^2}} =$$

$\frac{1}{t} \to +\infty$

$$= \lim_{t\to 0} (-t) = 0.$$

<u>Problem:</u> $\lim_{(x,y)\to(0,0)} (x^2+y^2)^{x^2+y^2}$

<u>Solution</u> $t := x^2+y^2$

$$\lim_{(x,y)\to(0,0)} (x^2+y^2)^{x^2+y^2} = \lim_{t\to 0} t^t =$$

$$= \lim_{t\to 0} \left(e^{\ln t}\right)^t = \lim_{t\to 0} e^{t\ln t}$$

Look at $\lim_{t\to 0} t\ln t = 0$ by previous problem.

$$= e^0 = 1. \quad \square$$

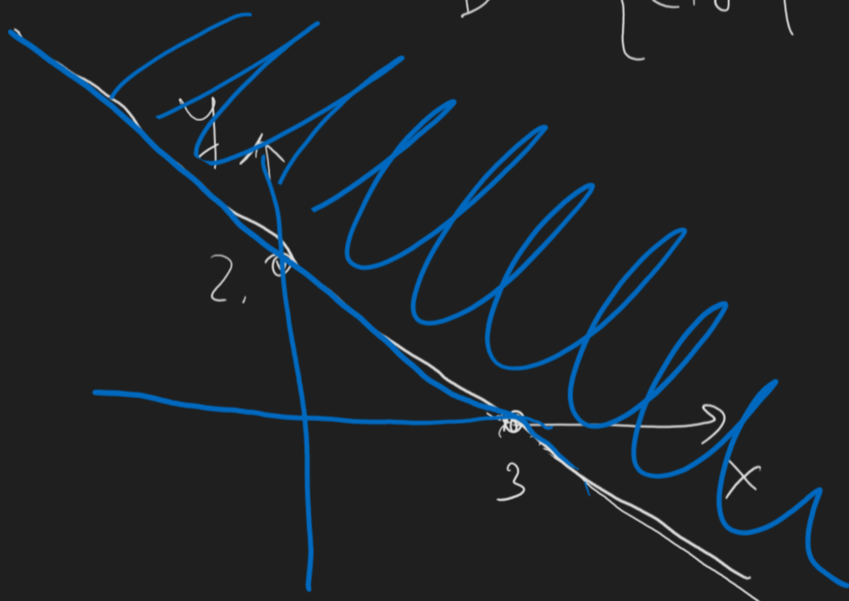
tion 4.2: 25 Find $h(x,y)=g(f(x,y))$ and the set of pts at which h is continuous, if

$$g(t)=t^2+\sqrt{t}, \quad f(x,y)=2x+3y-6$$

Solution $h(x,y)=g(f(x,y))=$

$$=g(2x+3y-6)=\boxed{(2x+3y-6)^2+\sqrt{2x+3y-6}}$$

By Continuity Laws h is contin. at its domain $D=\{(x,y)\mid 2x+3y-6\geq 0\}$.



$$2x+3y-6=0$$
$$3y=6-2x$$
$$y=2-\frac{2}{3}x$$

Section 14.3: 52 Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

a) $z=f(x)g(y)$

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TS

section 14.3 : 52 Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

a) $z = f(x)\,g(y)$

b) $z = f(xy)$

c) $z = f\left(\frac{x}{y}\right)$.

Solution a) $z = f(x)\,g(y)$.

$$\frac{\partial z}{\partial x} = f'(x)\,g(y).$$

$$\frac{\partial z}{\partial y} = f(x)\cdot g'(y).$$

b) $z = f(xy)$.

$$\frac{\partial z}{\partial x} = f'(xy)\cdot y.$$

$$\frac{\partial z}{\partial y} = f'(xy)\cdot x.$$

c) $z = f\left(\frac{x}{y}\right)$

$$\frac{\partial z}{\partial x} = f'\left(\frac{x}{y}\right)\cdot \frac{1}{y}$$

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c) $z = f\left(\frac{x}{y}\right)$

$$\frac{\partial z}{\partial x} = f'\left(\frac{x}{y}\right) \cdot \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = f'\left(\frac{x}{y}\right) \cdot \left(-\frac{x}{y^2}\right)$$

Problem: Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if

$$z = f(x+y) - g(y^3).$$

Solution $\frac{\partial z}{\partial x} = f'(x+y)$

$$\frac{\partial z}{\partial y} = f'(x+y) - g'(y^3) \cdot 3y^2.$$

$\square$.

Problem $f(x,y,z) =$

$$= z + yz \sin x + \sqrt{\ln(\cos(y^2 z))}$$

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Problem $f(x,y,z) =$

$$= z + yz\sin x + \sqrt{\ln(\cos(y^2 z))}$$

Find f_{zx}.

Solution $f_{zx} = (f_z)_x$

Let us use that $\boxed{f_{zx} = f_{xz}}$

(this is true under the assumption that f_{zx} and f_{xz} are contin. which is automatic in our case).

$$f_{zx} = f_{xz} = (f_x)_z$$

$$f_x = yz\cos x,$$

$$f_{xz} = y\cos x. \quad \square$$