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sect. 14.5: 1, 9, 22, 27, 31

1: $z = xy^3 - x^2 y$, $x = t^2+1$, $y = t^2 - 1$.

Find $\frac{dz}{dt}$.

Solution $\boxed{\text{Apply Chain Rule (version 1):}}$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} =$$

$$= (y^3 - 2xy) \cdot 2t + (3xy^2 - x^2) \cdot 2t. \quad \square$$

9: Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

$z = \ln(3x+2y)$, $x = s \sin t$, $y = t \cos s$.

Solution Apply Chain Rule (version 2):

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} =$$

$$= \frac{3}{3x+2y} \cdot \sin t + \frac{2}{3x+2y} \cdot (-t \sin s)$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} =$$

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$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} =$$

$$= \frac{3}{3x+2y} \cdot s\cos t + \frac{2}{3x+2y} \cdot \cos s.$$

$\square$.

22: $u = \sqrt{r^2+s^2}$, $\boxed{r = y + x\cdot\cos t}$, $s = x + y\cdot\sin t$

Find $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial u}{\partial t}$ when $\boxed{x=1, y=2, t=0.}$

Solution u depends of r, s

r, s depend on x, y, t.

Apply chain Rule (general version):

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial x} + \frac{\partial u}{\partial s}\cdot\frac{\partial s}{\partial x} =$$

$$= \frac{\cancel{2}r}{\cancel{2}\sqrt{r^2+s^2}}\cdot\cos t + \frac{\cancel{2}s}{\cancel{2}\sqrt{r^2+s^2}}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial y} + \frac{\partial u}{\partial s}\cdot\frac{\partial s}{\partial y} = \frac{r}{\sqrt{r^2+s^2}} + \frac{s}{\sqrt{r^2+s^2}}\cdot\sin t$$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial t} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial t} = \frac{r}{\sqrt{r^2+s^2}}\cdot(-x\sin t) +$$

$$+ \frac{s}{\sqrt{r^2+s^2}}\cdot y\cos t$$

When $x=1, y=2, t=0$: $\frac{\partial u}{\partial x}(1,2,0) =$

$$= \frac{r}{\sqrt{r^2+s^2}}\cdot \cos t + \frac{s}{\sqrt{r^2+s^2}} \Bigg|_{x=1,\,y=2,\,t=0} =$$

when $x=1, y=2, t=0$ we have

$r = 2+1\cdot\cos 0 = 3$

$s = 1$

substitute

$$= \frac{3}{\sqrt{3^2+1^2}}\cdot \cos 0 + \frac{1}{\sqrt{3^2+1^2}} = \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{10}} = \frac{4}{\sqrt{10}}.$$

milarly evaluate

$\frac{\partial u}{\partial y}$ and $\frac{\partial u}{\partial t}$ at $x=1$, $y=2$, $t=0$.

: $y\cos x = x^2+y^2$. Find $\frac{dy}{dx}$.

tion y is a f-n of x given implicitly.

troduce $F(x,y) = y\cos x - x^2 - y^2$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{-y\sin x - 2x}{\cos x - 2y} = \frac{y\sin x + 2x}{\cos x - 2y}. \quad \square$$

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31: $x^2+2y^2+3z^2=1$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

Solution

$$F(x,y,z) = x^2+2y^2+3z^2-1.$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{2x}{6z} = -\frac{x}{3z}.$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{4y}{6z} = -\frac{2y}{3z}. \quad \square$$

True - false test : 1-6, 8, 11.

1: $f_y(a,b) = \lim_{y \to b} \frac{f(a,y)-f(a,b)}{y-b}$

True : $f_y(a,b) = \lim_{h \to 0} \frac{f(a,\cancel{b+h}^{\,y})-f(a,b)}{h} =$

$$= \lim_{y \to b} \frac{f(a,y)-f(a,b)}{y-b}.$$

2: There exists a f-n f with contin. 2^{nd} order partial derivatives such that

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2. There exists a f-n f with contin. 2^{nd} order partial derivatives such that

$$\underline{f_x = x + y^2}, \quad \underline{f_y = x - y^2}.$$

False: Suppose such f-n exists. Then

$$f_{xy} = f_{yx}.$$

$$f_{xy} = (x+y^2)_y = 2y. \quad \neq$$

$$f_{yx} = (x-y^2)_x = 1$$

Hence such $f \nexists$.

3: $$f_{xy} = \frac{\partial^2 f}{\partial x \partial y}$$

this would be correct.

False: $$f_{xy} = (f_x)_y = \frac{\partial^2 f}{\partial y \partial x}$$

$$\boxed{\frac{\partial^2 f}{\partial x \partial y}} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = (f_y)_x = f_{yx}.$$

4: $$D_{\vec{k}} f^{(x,y,z)} = f_z^{(x,y,z)}.$$

True: $D_{\bar{k}} f = \nabla f \cdot \bar{k} = \langle f_x, f_y, f_z \rangle \cdot \langle 0, 0, 1 \rangle = f_z$.

5: If $f(x,y) \to L$ as $(x,y) \to (a,b)$ along every straight line through (a,b) then $\lim\limits_{(x,y)\to(a,b)} f(x,y) = L$.

False. We had such example

② $f(x,y) = \frac{xy^2}{x^2+y^4}$.

We showed: along any straight line $f(x,y) \to 0$.
along parabola $x = y^2$: $f(x,y) \to 1/2$.

6: If $f_x(a,b)$ and $f_y(a,b)$ both exist, then f is differentiable at (a,b).

False: we had such example

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y) \neq (0,0). \\ 0, & (x,y) = (0,0) \end{cases}$$

We showed $f_x(0,0)$ and $f_y(0,0)$ $\exists$,
but f is not differentiable.

We showed $f_x(0,0)$ and $f_y(0,0)$

but f is not differentiable.

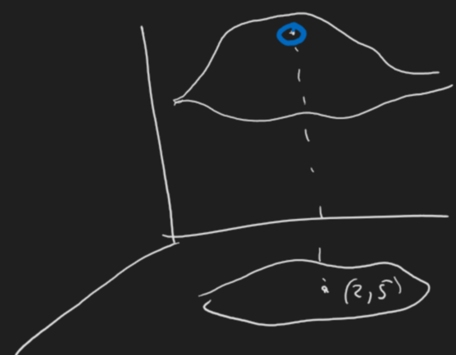
However if f_x and f_y are contin., then f is different.

8: If f is a function, then

$$\boxed{\lim_{(x,y)\to(2,5)} f(x,y) = f(2,5).}$$

False

Would be true if f was continuous at $(2,5)$



11: If $f(x,y) = \sin x + \sin y$, then

$$-\sqrt{2} \leq D_{\bar u} f(x,y) \leq \sqrt{2}.$$

True: $D_{\bar u} f = \nabla f \cdot \bar u =$

$= |\nabla f| \cdot |\bar u| \cdot \cos\alpha$

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$-\sqrt{2} \le D_{\bar{u}} f(x,y) \le \sqrt{2}$

True:

$$D_{\bar{u}} f = \nabla f \cdot \bar{u} =$$

$$= |\nabla f| \cdot |\bar{u}| \cdot \cos \alpha$$

$|\bar{u}| = 1$; α: angle between ∇f and $\bar{u}$.

$$\underline{D_{\bar{u}} f = |\nabla f| \cdot \cos \alpha}$$

$$-1 \le \cos \alpha \le 1$$

$$-|\nabla f| \le D_{\bar{u}} f \le \underline{|\nabla f|}$$

$$\nabla f = \langle \cos x, \cos y \rangle$$

$$\text{so } |\nabla f| = \sqrt{\cos^2 x + \cos^2 y} \le \underline{\underline{\sqrt{2}}}.$$

$\cos^2 x \le 1$, $\cos^2 y \le 1$

$$-\sqrt{2} \le D_{\bar{u}} f \le \sqrt{2}. \qquad \square.$$

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