15.6.13


compute $A = \iiint_E 6xy \, dV$

E lies under the plane

(1) $\boxed{z = 1+x+y}$ and above

the region in the xy-plane

bounded by the curves

$y = \sqrt{x}$ and $x = 1$

(1) $0 = x + y - (z-1)$

→ one orthogonal vector is $(1, 1, -1)$

→ one vector in this plan is $(0, 1, 1)$

→ an other vector in this plan not colinear to the first one is $(1, -1, 0)$

correction of last week

14.8.23 $\nabla f = \lambda \nabla g \Rightarrow \begin{cases} -ye^{-xy} = 2\lambda x \\ -xe^{-xy} = 8\lambda y \end{cases}$

→ $\min = e^{-1/4}$

$\max = e^{+1/4}$ on $\left(\pm \frac{\sqrt{2}}{2}, \pm \frac{\sqrt{2}}{4}\right)$

14.8.25

b) $\nabla f(0,0) = (1, 0)$ ⇒ $\nexists \lambda \in \mathbb{R}$

$\nabla g(0,0) = (0,0)$

$\nabla f(0,0) = \lambda \nabla g(0,0)$

$$A = \iiint_E 6xy\,dV = \int_0^1 \int_0^{\sqrt{x}} \int_0^{1+x+y} 6xy\,dz\,dy\,dx$$

$$= \int_0^1 \int_0^{\sqrt{x}} 6xy\,(1+x+y)\,dy\,dx$$

$$= \int_0^1 6x\left((1+x)\left[\frac{y^2}{2}\right]_0^{\sqrt{x}} + \left[\frac{y^3}{3}\right]_0^{\sqrt{x}}\right)dx$$

$$= \int_0^1 3x^2(1+x) + 2x^{5/2}\,dx$$

$$= 3\left[\frac{x^3}{3}\right]_0^1 + 3\left[\frac{x^4}{4}\right]_0^1 + 2\left[\frac{2}{7}x^{7/2}\right]_0^1$$

$$= 1 + \frac{3}{4} + \frac{4}{7} = \frac{28+21+16}{28} = \frac{65}{28}$$

15.6.9 $A = \iiint_E y \, dV \qquad E = \{ (x, y, z) \in \mathbb{R}^3 , \; 0 \leq x \leq 3 , \; 0 \leq y \leq x , \; x - y \leq z \leq x + y \}$

$$= \int_0^3 \int_0^x \int_{x-y}^{x+y} y \, dz \, dy \, dx$$

$$= \int_0^3 \int_0^x y \times 2y \, dy \, dx$$

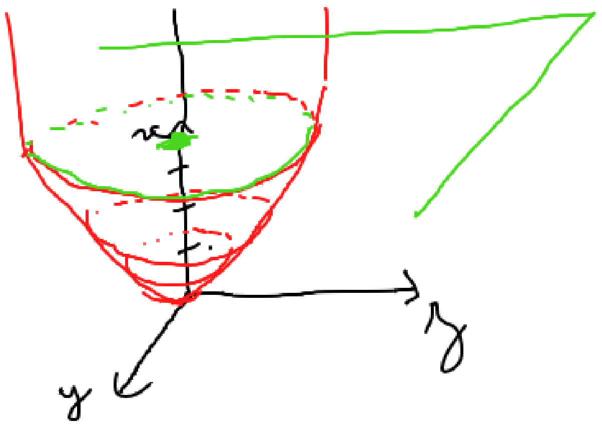
$$= \int_0^3 2 \left[ \frac{y^3}{3} \right]_0^x dx$$

$$= \frac{2}{3} \left[ \frac{x^4}{4} \right]_0^3$$

$$= \frac{27}{2}$$

15.6.17

$A = \iiint_E x \, dV \qquad E$ between $x = 4y^2 + 4z^2$ and $x = 4$




$A = \int_0^4 \int_{D_x} x \, dy\, dz \, dx$ with $D_x$ = disk of radius $\frac{\sqrt{x}}{2}$

$= \int_0^4 x \underbrace{\int_{D_x} dy\, dz}_{B = \text{Area of } D_x = \pi \left(\frac{\sqrt{x}}{2}\right)^2 = \frac{\pi x}{4} \ (*)} dx \qquad D_x = \{(\frac{\sqrt{x}}{2}, y, z) \mid , \ y^2 + z^2 \leq \frac{x}{4}\}$

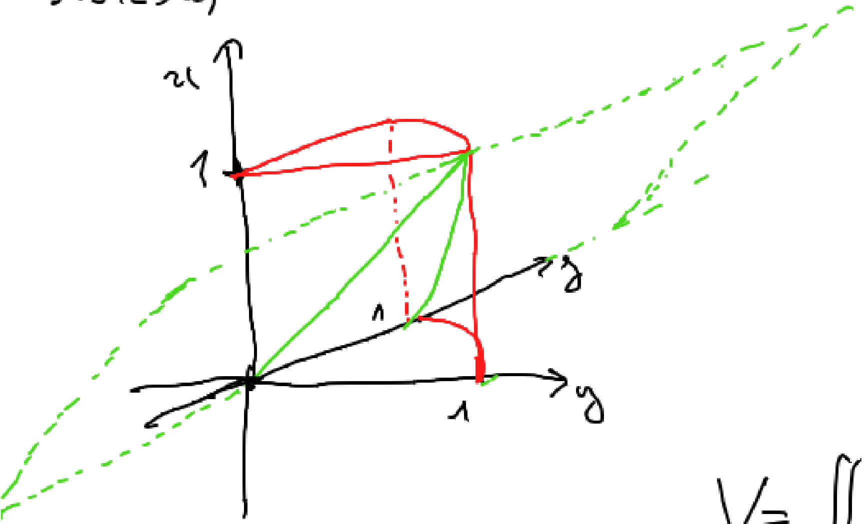
$= \int_0^4 \frac{\pi}{4} x^2 \, dx$

$= \frac{\pi}{4} \left[\frac{x^3}{3}\right]_0^4 = \frac{16\pi}{3}$

$(*)\ B = \int_0^{\frac{\sqrt{x}}{2}} \int_0^{2\pi} r \, d\theta \, dr = \int_0^{\frac{\sqrt{x}}{2}} r \, 2\pi \, dr = 2\pi \left[\frac{r^2}{2}\right]_0^{\frac{\sqrt{x}}{2}} = \pi \frac{x}{4}$

(polar coordinates)

15.6.23 a)




the volume of the wedge in the first octant cut from the cylinder $y^2+z^2=1$ (1)

and $y=x$ and $x=1$

first octant: $x \geq 0,\ y \geq 0,\ z \geq 0$

$$V=\iiint_E dV = \int_0^1 \int_0^x \int_0^{\sqrt{1-y^2}} dz\,dy\,dx$$

* with x and y fixed, z is smaller than the value given by (1)

$z \leq \sqrt{1-y^2}$ ( $z \geq 0$ because we look at the 1st octant)

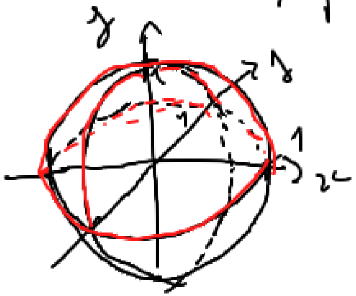
15.6.54

recall $f:\mathbb{R}^3 \to \mathbb{R}$ then the average value on $E \subset \mathbb{R}^3$ is

$$f_{ave} = \frac{1}{V(E)} \iiint_E f \, dV$$

Find the average height of the point in the solid hemisphere

$x^2+y^2+z^2 \le 1$ $\quad z \ge 0$




$$z_{ave} = \frac{1}{V(E)} \iiint_E z \, dV \qquad V(E) = \frac{1}{2}(4\pi 1^3) = 2\pi$$

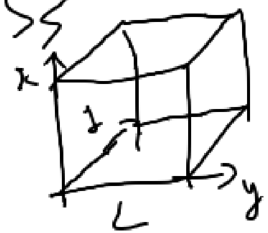
$$= \frac{1}{2\pi} \int_0^1 \int_{D_z} z \, dx\,dy\,dz \quad \text{with } D_z = \{(x,y,z),\ x^2+y^2 \le 1-z^2\}$$

Area of $D_z = \pi\left(\sqrt{1-z^2}\right)^2$

$$= \frac{1}{2\pi} \int_0^1 z \int_{D_z} dx\,dy \, dz = \frac{1}{2\pi} \int_0^1 z\pi(1-z^2)\,dz = \frac{1}{2}\left(\left[\frac{z^2}{2}\right]_0^1 - \left[\frac{z^4}{4}\right]_0^1\right)$$

so $z_{ave} = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{1}{8}$

15.6.53




$$\rho_{ave} = \frac{1}{V(E)} \iiint_E xyz \, dV$$

$$= \frac{1}{L^3} \int_0^L \int_0^L \int_0^L xyz \, dx \, dy \, dz$$

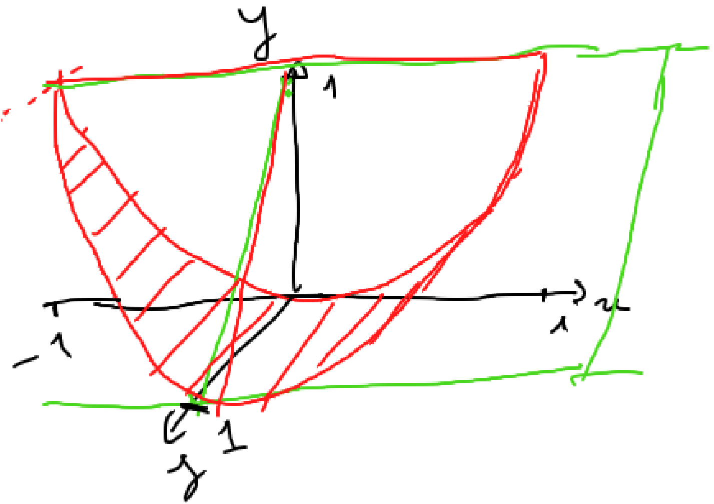
$$= \frac{1}{L^3} \left( \int_0^L x \, dx \right)^3$$

$$= \frac{1}{L^3} \left( \left[ \frac{x^2}{2} \right]_0^L \right)^3 = \frac{1}{L^3} \frac{1}{8} L^6$$

$$= \frac{L^3}{8}$$

15.6.21

the volume inside the cylinder $y = x^2$ and the plane $z = 0$ $y + z = 1$




$$V = \iiint_E dV$$

$$= \int_{-1}^{1} \int_{x^2}^{1} \int_{0}^{1-y} dz\, dy\, dx$$

$$= \int_{-1}^{1} \int_{x^2}^{1} (1-y)\, dy\, dx$$

$$= \int_{-1}^{1} (1 - x^2) - \left[\frac{y^2}{2}\right]_{x^2}^{1} dx$$

$$= 2 - \left[\frac{x^3}{3}\right]_{-1}^{1} - \frac{1}{2} \times 2 + \frac{1}{2}\left[\frac{x^5}{5}\right]_{-1}^{1}$$

$$= 2 - \frac{2}{3} - 1 + \frac{1}{2}\,\frac{2}{5} = \frac{15 - 10 + 3}{15} = \frac{8}{15}$$