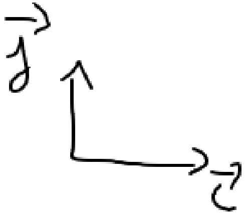
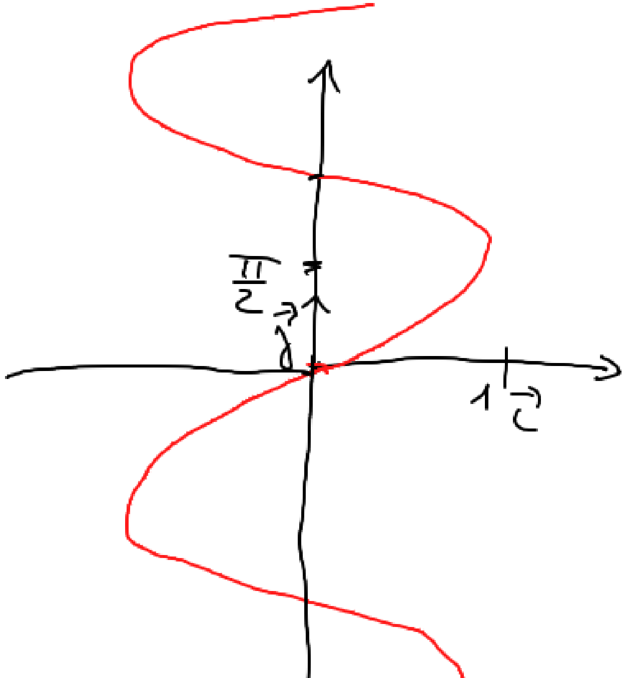
13.1.7

sketch $r(t) = \langle \sin(t), t \rangle$

$= \sin(t)\vec{i} + t\,\vec{j}$



Rk $y = f(x)$ $\quad r(x) = \langle x, f(x) \rangle$

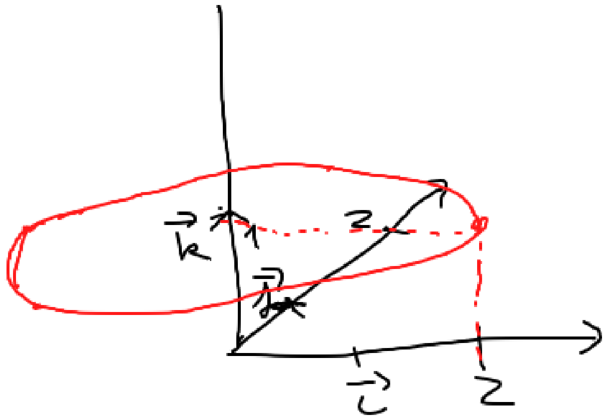


13.1.12 sketch $r(t) = 2\cos t\,\vec{i} + 2\sin t\,\vec{j} + \vec{k}$

$$= \langle 2\cos t, 2\sin t, 1 \rangle$$

$$= 2(\cos t\,\vec{i} + \sin t\,\vec{j}) + \vec{k}$$

$$= 2\langle \cos t, \sin t, 0 \rangle + \langle 0, 0, 1 \rangle$$



Rk: $f(t) = \langle \overset{x}{\cos(t)}, \overset{y}{\sin(t)} \rangle$

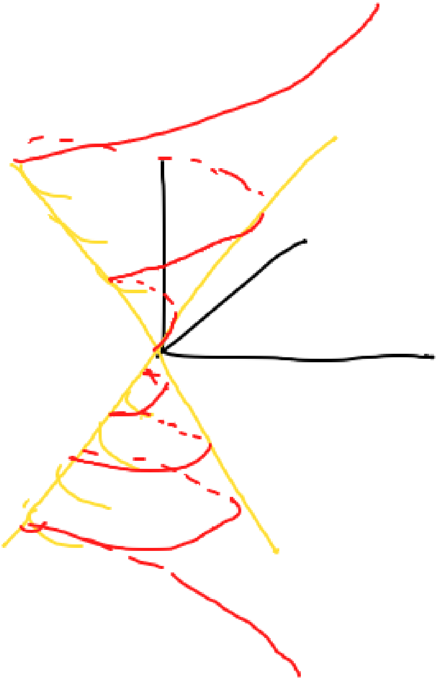
$$x^2 + y^2 = \cos^2(t) + \sin^2(t) = 1 \ \forall t$$

13.1.27 Show that $x = t\cos t$, $y = t\sin t$, $z = t$ lies on the cone $C: z^2 = x^2+y^2$

1) $(\underbrace{t\cos t}_{x})^2 + (\underbrace{t\sin t}_{y})^2 = \underbrace{t}_{z}^2 \times 1 \quad \Leftrightarrow \quad x^2+y^2=z^2$

So for all $t \in \mathbb{R}$ $r(t) \in C$

2)



13.1.49

will $r_1(t) = \langle t^2, 7t-12, t^2 \rangle$ and $r_2(t) = \langle 4t-3, t^2, 5t-6 \rangle$ collide ?

i.e $\exists\, t_0 \in \mathbb{R}, \quad r_1(t_0) = r_2(t_0)$?

$$r_1(t) = r_2(t) \Rightarrow \begin{cases} t^2 = 4t-3 \\ 7t-12 = t^2 \\ t^2 = 5t-6 \end{cases} \Rightarrow \begin{cases} 7t-12 = 4t-3 \\ \vdots \\ \vdots \end{cases} \Rightarrow \begin{cases} 3t = 9 \\ \vdots \\ \vdots \end{cases} \Rightarrow \begin{cases} t = 3 \\ \\ \end{cases}$$

for $t=3$ we verify that $r_1(3) = r_2(3)$

So there will be a collision

13.2.25

find a parametric equation for the tangent line of
$x = e^{-t}\cos t$, $y = e^{-t}\sin t$, $z = e^{-t}$ at the point $(1, 0, 1)$

Rk $y = f(x)$ the tangent line at x_0 given by $y = f'(x_0)x + f(x_0)$

a) $r'(t) = -e^{-t} < \cos t, \sin t, 1 > + e^{-t} < -\sin t, \cos t, 0 >$

$= e^{-t} < -(\cos t + \sin t), \cos t - \sin t, -1 >$

b) we look for t such that $r(t) = (1, 0, 1) \Rightarrow \begin{cases} e^{-t}\cos t = 1 \\ e^{-t}\sin t = 0 \\ e^{-t} = 1 \end{cases} \Rightarrow \begin{cases} 1 = 0 \\ 0 = 0 \\ t = 0 \end{cases}$

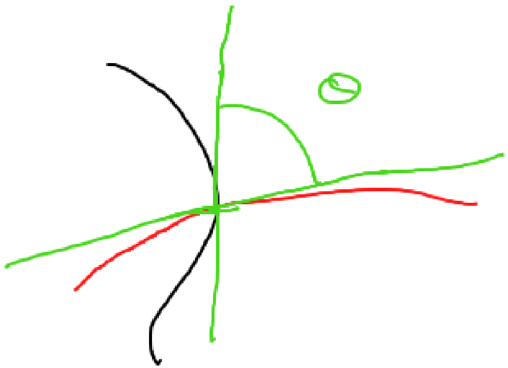
for $t = 0$ we have $r(0) = (1, 0, 1)$

c) $r'(0) = (-1, 1, -1)$

d) $T(t) = r(0) + t\, r'(0) = < 1-t, t, 1-t >$

13.2.33 find the angle of intersection at the origin of

$r_1(t) = \langle t, t^2, t^3 \rangle$ $\quad r_2(t) = \langle \sin t, \sin 2t, t \rangle$



1) $r_1'(t) = \langle 1, 2t, 3t^2 \rangle$ $\quad r_1'(0) = \langle 1, 0, 0 \rangle$

$r_2'(t) = \langle \cos t, 2\cos 2t, 1 \rangle$ $\quad r_2'(0) = \langle 1, 2, 1 \rangle$

2) $\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$

$\|r_1'(0)\| = \sqrt{1^2 + 0^2 + 0^2} = 1$ $\quad \|r_2'(0)\| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$

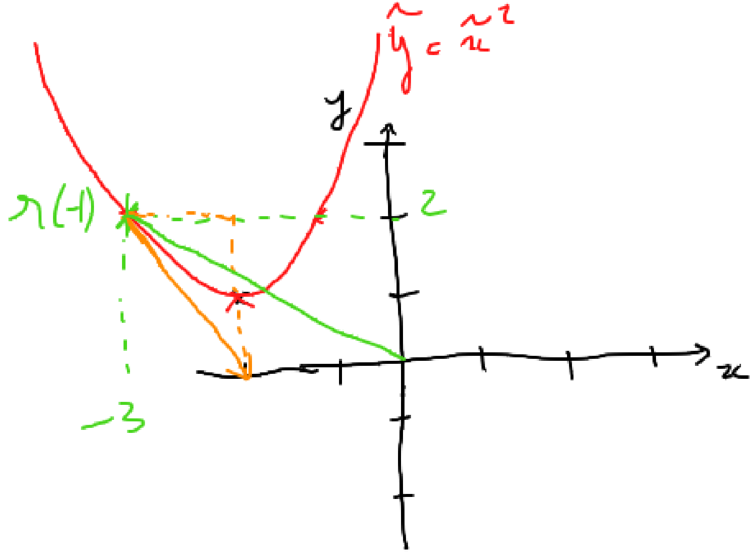
$$\frac{r_1'(0)}{\|r_1'(0)\|} \cdot \frac{r_2'(0)}{\|r_2'(0)\|} = 1 \cdot \frac{1}{\sqrt{6}} + 0 \cdot \frac{2}{\sqrt{6}} + 0 \cdot \frac{1}{\sqrt{6}} = \frac{1}{\sqrt{6}} = \cos\theta$$

3) $\theta = \arccos\left(\frac{1}{\sqrt{6}}\right) \approx 66^\circ$

13.2.3 a) sketch the curve b) find $r'(t)$ c) sketch $r(t)$ and $r'(t)$ at a point

a) $r(t) = < t-2, t^2+1 >$ at the point $t=-1$

$= <-2, 1> + <t, t^2>$



b) $r'(t) = < 1, 2t >$

c) $r'(-1) = < 1, -2 >$

$r(-1) = < -3, 2 >$

13.1.43 find a vector function for the curve defined by the intersection of two surfaces

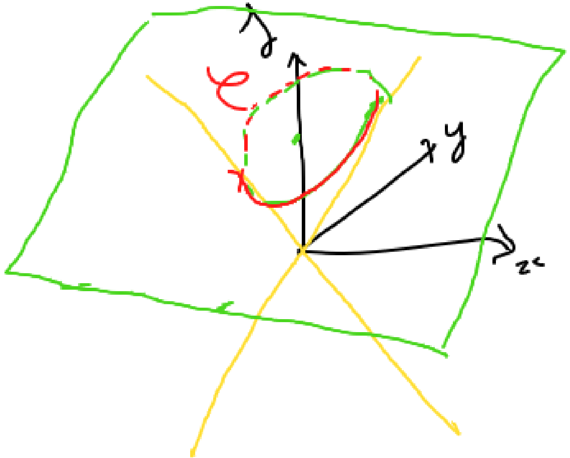
the cone $z = \sqrt{x^2+y^2}$ and the plane $z = 1+y$

$$\mathcal{C} : \{ (x, y, z) \in \mathbb{R}^3 , \ z = \sqrt{x^2+y^2} \text{ and } z = 1+y \}$$

so $(1+y)^2 = x^2+y^2 \Rightarrow 1+2y = x^2 \Rightarrow y = \dfrac{x^2-1}{2}$

$$\mathcal{C} : \left\{ \left(x, \frac{x^2-1}{2}, 1 + \frac{x^2-1}{2} \right), \ x \in \mathbb{R} \right\}$$

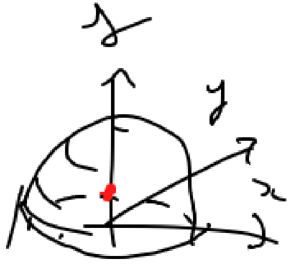
$$\therefore \ r(t) = \left\langle t, \frac{t^2-1}{2}, \frac{t^2+1}{2} \right\rangle$$



15.6.39 Rk: $M = \iiint_E \rho(x,y,z)\,dV$

moments $M_{yz} = \iiint_E x\,\rho(x,y,z)\,dV$

center of mass point with coordinate $\bar{x} = \dfrac{M_{yz}}{m}$



$\rho(x,y,z) = 3$ E lies between xy-plane and $z = 1 - x^2 - y^2$

$$M = \iiint_E \rho\,dV = 3 \times V = 3\int_0^1 \underbrace{\iint_{D_z} dx\,dy}_{\text{Area of } D_z \,=\, \pi\left(\sqrt{1-z}\right)^2 \,=\, \pi(1-z)}\,dz$$

$$= 3\pi\int_0^1 1 - z\,dz = 3\pi\left(1 - \left[\frac{z^2}{2}\right]_0^1\right) = \frac{3\pi}{2}$$

$$M\bar{z} = \iiint_E z\,\rho\,dV = 3\int_0^1 \iint_{D_z} z\,dx\,dy\,dz = 3\int_0^1 z\,\pi(1-z)\,dz = 3\pi\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{\pi}{2}$$

$$\bar{z} = \frac{\frac{\pi}{2}}{\frac{3\pi}{2}} = \frac{1}{3}$$

$\bar{x} = \bar{y} = 0$ because of the symmetry of the solid

center of mass $= (0, 0, 1/3)$

15.6. 41 on a cube $0 \le x, y, z \le a$ $\rho(x,y,z) = x^2+y^2+z^2$

$$M = \int_0^a\int_0^a\int_0^a x^2+y^2+z^2\,dx\,dy\,dz = \int_0^a\int_0^a \left[\int_0^a x^2\,dx + (y^2+z^2)\int_0^a dx\right] dy\,dz$$

$$= 3a^2\int_0^a x^2\,dx = 3a^2\,\frac{a^3}{3} = a^5$$

$$M_{yz} = \iiint_E x(x^2+y^2+z^2)\,dV = \int_0^a x\left(a^2x^2 + a\int_0^a y^2\,dy + a\int_0^a z^2\,dz\right)dx$$

$$= a^2\,\frac{a^4}{4} + \int_0^a x\,\frac{2a^4}{3}\,dx = \frac{a^6}{4} + \frac{2a^4}{3}\,\frac{a^2}{2} = \frac{7}{12}a^6 = M_{xz} = M_{xy}$$

$$\bar{x} = \frac{M_{yz}}{M} = \frac{7}{12}a = \bar{y} = \bar{z}$$

center of mass $= \left(\frac{7}{12}a, \frac{7}{12}a, \frac{7}{12}a\right)$