

Index

- Abadie's CQ, 141
active constraint ($\mathcal{I}(\mathbf{x})$), 95
adjacent extreme point, 235
affine independence, 37
affine function, 14, 66
affine subspace, 36
affine transformation, 433
algebraic characterization of adjacency, 236
approximate line search, 300
Armijo step, 301, 332, 431
artificial variables, 247
augmented Lagrangian function, 441
augmented Lagrangian method, 441
automatic differentiation, 322
- Banach's Theorem, 117
barrier function, 378
barrier problem, 379
basic feasible solution, 229
basic solution, 229
basic variables, 229
basis, 37
BFGS method, 296
Bland's rule, 252
boundary, 40
bounded set, 40
Brouwer's Theorem, 117
bundle method, 182
- calculus rules, 42
canonical form, 257
Carathéodory's Theorem, 48
Cartesian product set, 164
Cauchy–Bunyakowski–Schwarz inequality, 36
- central difference formula, 321
centralized planning, 285
classification of optimization models, 14
closed sets, 40
closed mapping, 175, 344
closure, 40
coercive function, 84
column dropping, 329
column generation, 9
combinatorial optimization, 201
complementarity, 164
Complementary Slackness Theorem, 264
composite function, 67, 121
composite operator, 121
concave function, 66
cone, 53
 - polar, 114
 - polyhedral, 53cone of feasible directions, 129
conjugate direction, 310, 319
conjugate gradient, 313
conjugate gradient method, 313
constrained optimization, 14, 94–103, 323–396, 436–441
constraint qualification (CQ), 19, 140, 141, 147, 148
constraints, 5
continuity, 103
continuous function, 40
continuous optimization, 14
continuous relaxation, 201
continuously differentiable function, 41

Index

- contractive operator, 117
convergence rate, 320
 geometric, 117, 186
 linear, 320
 quadratic, 321
 superlinear, 320
convex problem, 150
convex analysis, 43–77, 402–405
convex function, 65, 103, 175
convex problem, 82, 160, 165, 181
convex programming, 15
convex set, 43
coordinates, 37
CQ, 140

Danskin’s Theorem, 176
Dantzig–Wolfe algorithm, 174
decision science, 12
decision variable, 8
degenerate basic solution, 229
descent direction, 92, 181
descent lemma, 436
DFP method, 317
Diet problem, 13
differentiability, 179
differentiable function, 41
differentiable optimization, 15
Dijkstra’s algorithm, 340
diode, 198
direction of unboundedness, 240
directional derivative, 40, 93, 175
distance function, 75
divergent series step length rule, 181,
 328
domination, 394
dual feasible basis, 269
dual infeasible basis, 269
dual linear program, 256
dual simplex algorithm, 270
dual simplex method, 268
duality gap, 161

effective domain, 103, 160
efficient frontier, 394
eigenvalue, 39
eigenvector, 39
Ekeland’s variational principle, 410

electrical circuit, 197
electrical network, 197
eligible entering variable, 252
eligible leaving variable, 252
epigraph, 69, 86
 ε -optimal solution, 103
equality constraint, 14
equivalent systems, 423
Euclidean projection, 74
Everett’s Theorem, 193
exact penalty function, 388
existence of optimal solution, 233
exterior penalty method, 374–378
extreme direction, 232
extreme point, 49

Farkas’ Lemma, 60, 105, 109, 155,
 263
Farkas’ Lemma for an inequality
 system, 60
feasibility heuristic, 204
feasible direction, 95
feasible solution, 7, 16
feasible-direction methods, 323
filter, 394
filter-SQP methods, 394
finite termination, 305
finitely generated cone, 63
fixed point, 116
Fletcher–Reeves formula, 316
forward difference formula, 321
Fourier elimination, 56
Frank–Wolfe algorithm, 325
Frank–Wolfe Theorem, 88
Fritz John conditions, 136
full rank, 38

Gauss–Newton method, 297–299
Gauss–Seidel method, 83
Gauss–Seidel method, 121
geometric convergence rate, 117, 186
global minimum, 82
global optimality conditions, 150
global optimality conditions, 163
global optimum, 82
 necessary and sufficient con-
 ditions, 94, 97

Index

- sufficient conditions, 150
- Golden section, 301
- Gordan's Theorem, 62, 113
- gradient, 41
- gradient projection algorithm, 332
- gradient related, 293
- gradient related method, 303, 304
- Gram–Schmidt procedure, 313
- hard constraint, 20
- Hessian matrix, 41
- hyperplane
 - proper supporting, 110
 - supporting, 110
- $\mathcal{I}(\mathbf{x})$, 95
- identity matrix \mathbf{I}^n , 39
- ill-conditioning, 394
- implicit function, 42, 321
- Implicit Function Theorem, 180
- indicator function (χ_S), 183, 373
- inequality constraint, 14
- infimum, 17
- infinite-dimensional optimization, 16
- integer programming, 14, 15
- integrable function, 338
- integrality property, 15
- interior, 40
 - interior penalty function, 123
 - interior point algorithm, 253
 - interior point algorithm, 378–385
 - interpolation, 301
 - iso-cost line, 290
 - iso-curve, 290
- Jacobi method, 121
- Jacobian, 41, 338, 387, 432
- Karmarkar's algorithm, 253
- Karush–Kuhn–Tucker (KKT) conditions, 142, 150
- kernel, 38
- Kirchhoff's laws, 198
- Lagrange function, 158
- Lagrange multiplier method, 174, 190
- Lagrange multiplier vector, 159, 196
- Lagrange multipliers, 137
- Lagrangian relaxation, 21
- Lagrangian dual function, 159
- Lagrangian dual problem, 159
- Lagrangian duality, 157–208, 417–421
- Lagrangian relaxation, 158, 159, 201
- least-squares data fitting, 289
- least-squares problems, 297–299
- level curve, 290
- level set ($\text{lev}_g(b)$), 73, 74, 85, 86, 168, 183, 303, 304, 332
- Levenberg–Marquardt, 295, 433
- LICQ, 148
- limit, 39
- limit points, 40
- line search, 299
 - approximate, 300
 - Armijo step length rule, 332
 - Armijo step length rule, 301, 431
 - Golden section, 301
 - interpolation, 301
 - Newton's method, 301
- linear programming, 14
- linear convergence rate, 320
- linear function, 42
- linear independence, 37
- linear programming, 16, 170, 211–285, 384–385, 430
- linear programming duality, 255–285, 426–430
- linear space, 36
- linear subspace, 362
- linear-fractional programming, 422
- Lipschitz continuity, 304
- local minimum, 82
- local convergence, 387
- local optimum, 82
 - necessary conditions, 91, 93, 96, 132, 136, 142, 146
 - sufficient conditions, 93
- logarithmic barrier, 379
- logical constraint, 5, 7
- lower semi-continuity, 85

Index

- Maratos effect, 392
mathematical model, 4
mathematical programming, 11
matrix, 37
matrix game, 121
matrix inverse, 38
matrix norm, 37
matrix product, 38
matrix transpose, 37
max function, 176
mean-value theorem, 41
merit function, 389
method of successive averages (MSA), 371
MFCQ, 147
minimax theorem, 121
minimum, 17
minimum distance (dist_S), 183
Minkowski–Weyl Theorem, 64, 109
multi-objective optimization, 16, 394
near-optimality, 102
negative curvature, 292
neighbourhood, 40
Newton's method, 294, 301, 432
Newton–Raphson method, 121, 294
Nobel laureates, 13
non-basic variables, 229
non-convex programming, 15
non-coordinability, 192
non-differentiable function, 307
non-differentiable optimization, 15
non-expansive operator, 115
nonlinear programming, 14, 16
nonsingular matrix, 38
nonsingular matrix, 38
norm, 36
normal cone (N_x), 100, 142
NP-hard problem, 156, 202
null space, 38
objective function, 4
Ohm's law, 199
open ball, 40
open set, 40
operations research, 12
optimal BFS, 240
optimal solution, 7
optimal value, 7
optimality, 12
optimality conditions, 91–94, 96–101, 125–156, 163–165, 191–192, 241, 242, 266–267, 412–416
optimization under uncertainty, 16
optimize, 3
orthogonality, 36, 164
orthonormal basis, 37
parametric optimization, 154
Pareto optimal solution, 16
Pareto set, 394
partial pricing, 244
pattern search methods, 322
penalty, 21
penalty function, 21
penalty parameter, 374
perturbation function ($p(\mathbf{u})$), 194
Phase I, 336
phase I problem, 247
phase II problem, 248
physical constraint, 5, 7
piece-wise linear function, 307
Polak–Ribi  re formula, 316
polar cone, 114
Polyak step, 181
polyhedral cone, 53
polyhedron, 50
polytope, 49
positive (semi)definite matrix, 39
potential, 198
potential difference, 198
pre-conditioning, 316
price-directive decomposition, 282
primal infeasibility criterion, 269
primal simplex method, 268
projection, 74
projection matrix, 362
projection arc, 332
projection matrix, 362
projection operator, 74, 98, 115
projection problem, 336
proof by contradiction, 35

Index

- proper separation, 105
- proper function, 18, 183
- proper supporting hyperplane, 110
- proximal point algorithm, 434
- pseudo-convex function, 409
- Q -orthogonal**, 310
- quadratic programming, 172
- quadratic convergence rate, 321
- quadratic function, 42, 72, 156
- quadratic programming, 156, 336
- quasi-convex function, 410
- quasi-Newton methods, 295, 317, 390
- Rademacher's Theorem, 307
- range space, 38
- rank, 38
- rank-two update, 317
- recession cone, 88
- reduced gradient, 367
- reduced cost, 241
- redundant constraint, 408
- regular vector, 19
- relaxation, 21, 157, 158, 201, 376
- Relaxation Theorem, 157
- Representation Theorem, 328
- Representation Theorem, 53, 232
- resistor, 198
- restricted master problem, 329
- restricted simplicial decomposition, 330
- restification, 440
- revised simplex method, 253
- saddle point, 121, 122, 163, 292
- scalar product, 36
- secant method, 296
- sensitivity analysis, 154, 194, 196
- sensitivity analysis for LP, 272–276
- separation, 105
 - proper, 105
 - strict, 105
 - strong, 105
- Separation Theorem, 105, 179
- sequential linear programming (SLP), 440
- sequential quadratic programming (SQP), 385–394
- shadow price, 263
- shortest route, 340
- simplex method, 12, 239–254, 424–425
- simplicial decomposition algorithm, 328
- slack variable, 9
- Slater CQ, 148
- SLP algorithm, 440
- soft constraint, 21, 193
- solution set, 346
- spacer step, 350
- spectral theorem, 153
- SQP algorithm, 385–396, 441
- square matrix, 38
- stalling, 253
- standard basis, 37
- stationary point, 19
- stationary point, 91, 97
- steepest descent, 291
- steepest-edge rule, 244
- stochastic programming, 16
- strict separation, 105
- strict inequality, 87
- strict local minimum, 82
- strictly convex function, 66
- strictly quasi-convex function, 301
- strong separation, 105
- strong duality, 165
- Strong Duality Theorem, 165
- Strong Duality Theorem, 172
- Strong Duality Theorem, 168–170, 262
 - convex program, 165, 168, 169
 - linear program, 170, 262
 - quadratic program, 172
- subdifferentiability, 178
- subdifferential, 175
- subgradient, 174, 308
- subgradient optimization, 182
- subgradient projection method, 181
- superlinear convergence rate, 320
- supporting hyperplane, 110
- symmetric matrix, 39

Index

tangent cone, 129
traffic assignment problem, 339
traffic equilibrium, 337
triangle inequality, 38
trust region methods, 309
twice differentiable function, 41

unconstrained optimization, 14, 91–
 94, 289–322, 431–435
unimodal function, 301
unique optimum, 89
upper level set, 168
upper semi-continuity, 85
user equilibrium, 338

variable, 4
variational inequality, 97, 120
vector, 36
vector-valued functions, 41
voltage source, 198
von Neumann’s Minimax Theorem,
 121

Wardrop’s principle, 338
Weak Duality Theorem, 160, 261
 Lagrangian relaxation, 160
 linear program, 261
weak Wolfe condition, 302
weakly coercive function, 84
Weierstrass’ Theorem, 86, 105, 178
Wolfe condition, 302