$$= \frac{1}{(s+1)((s-1)^2+4)} + \frac{s-1}{(s-1)^2+4} + \frac{1}{(s-1)^2+4}$$

$$\frac{1}{(s+1)((s-1)^2+4)} \underset{\text{PBU}}{=} \frac{1}{8}\frac{1}{s+1} - \frac{1}{8}\frac{s-1}{(s-1)^2+4} + \frac{1}{4}\frac{1}{(s-1)^2+4}$$

$$\Rightarrow \tilde{u} = \frac{1}{8}\frac{1}{s+1} + \frac{7}{8}\frac{s-1}{(s-1)^2+4} + \frac{5}{4}\frac{1}{(s-1)^2+4}$$

$$\frac{1}{s+1} = \widetilde{e^{-t}}, \quad \frac{s}{s^2+4} = \widetilde{\cos 2t}, \quad \frac{s-1}{(s-1)^2+4} = \widetilde{e^{t}\cos 2t}$$

$$\frac{2}{s^2+4} = \widetilde{\sin 2t}, \quad \frac{2}{(s-1)^2+4} = \widetilde{e^{t}\sin 2t}$$

$$\Rightarrow \tilde{u} = \widetilde{\frac{1}{8}e^{-t} + \frac{7}{8}e^{t}\cos 2t + \frac{5}{8}e^{t}\sin 2t} \overset{}{\underset{\text{IF}}{\Longrightarrow}}$$

$$u = \frac{1}{8}\left(e^{-t} + 7e^{t}\cos 2t + 5e^{t}\sin 2t\right)$$

Kolla $u(0)=1$, $u'(0)=2$!

Z-transformen

Def: Om $\alpha := (a_k)_{k=0}^{\infty}$, $|a_k| \leq M r^k$, $M, r \in \mathbb{R}$,

då def. ZT: $\mathcal{Z}(\alpha)(z) := \sum_{k=0}^{\infty} \frac{a_k}{z^k}$

holo i $A(0, r, \infty)$

- $S(\alpha) := (a_{k+1})$ shift
- $\mathcal{Z}(S^n(\alpha)) = z^n\left(\mathcal{Z}(\alpha) - \sum_{0}^{n-1} \frac{a_k}{z^k}\right)$
- $\alpha * \beta := (c_n)$, $c_n := \sum_{j=0}^{k} a_j b_{k-j}$, faltning
- $\mathcal{Z}(\alpha * \beta) = \mathcal{Z}(\alpha)\mathcal{Z}(\beta)$

Ex: Hitta följden a_k, $k \geq 0$, som löser differensekv.

$$*: \quad a_{k+2} + a_k = b_{k+1} - b_k, \text{ där}$$

$a_0 = 0$, $a_1 = 1$, b_k given begr. följd, $b_0 = 0$.

Lösning: Låt $\alpha := (a_k)$, $\beta := (b_k)$. * kan

då skrives $S^2(\alpha) + \alpha = S(\beta) - \beta$. ZT av detta ger:

$$z^2\Big(Z(\alpha) - \underset{0}{\underset{\shortparallel}{a_0}} - \frac{\overset{1}{\overset{\shortparallel}{a_1}}}{z}\Big) + Z(\alpha) = z\big(Z(\beta) - \underset{0}{\underset{\shortparallel}{b_0}}\big) - Z(\beta)$$

$$\Rightarrow (z^2+1)Z(\alpha) - z = (z-1)Z(\beta) \Rightarrow$$

$$\Rightarrow \underline{Z(\alpha) = \frac{z-1}{z^2+1} Z(\beta) + \frac{z}{z^2+1}}$$

$$\frac{1}{z^2+1} = \frac{1}{z^2}\,\frac{1}{1-(-1/z^2)} \overset{|z|>1}{=} \frac{1}{z^2}\sum_0^\infty (-1/z^2)^k = \sum_0^\infty \frac{(-1)^k}{z^{2k+2}} =$$

$$= \frac{1}{z^2} - \frac{1}{z^4} + \frac{1}{z^6} - \dots$$

$$\underline{\frac{z-1}{z^2+1}} = \frac{z}{z^2+1} - \frac{1}{z^2+1} = \sum_0^\infty \frac{(-1)^k}{z^{2k+1}} - \sum_0^\infty \frac{(-1)^k}{z^{2k+2}} =$$

$$= \frac{1}{z} - \frac{1}{z^2} - \frac{1}{z^3} + \frac{1}{z^4} + \frac{1}{z^5} - \dots = \underline{\sum_0^\infty \frac{h_k}{z^k}}, \quad (h_k) = (0,1,-1,-1,1,1,-1,-1,\dots)$$

$$\frac{z}{z^2+1} = \sum_0^\infty \frac{(-1)^k}{z^{2k+1}} = \frac{1}{z} - \frac{1}{z^3} + \frac{1}{z^5} - \dots = \sum_0^\infty \frac{g_k}{z^k},$$

$$(g_k) = (0,1,0,-1,0,1,0,-1,\dots)$$

$$Z(\alpha) = Z((h_k))\,Z(\beta) + Z((g_k)) \overset{\text{Prop}}{=}$$

$$= Z\big(\underline{(h_k)*\beta + (g_k)}\big) \overset{\text{IF}}{\Rightarrow} \alpha = (h_k)*\beta + (g_k),$$

dvs $\underline{a_k = \sum_{j=0}^{k} h_j b_{k-j} + g_k}$.

Om säg $(b_k) = (2^k)$, $k \geq 1$, $b_0 = 0$. vad blir a_4?

$(0,2,4,8,16,\dots)$

Lösning: $a_4 = \underset{0}{\underset{\shortparallel}{h_0}}\underset{16}{\underset{\shortparallel}{b_4}} + \underset{1}{\underset{\shortparallel}{h_1}}\underset{8}{\underset{\shortparallel}{b_3}} + \underset{-1}{\underset{\shortparallel}{h_2}}\underset{4}{\underset{\shortparallel}{b_2}} + \underset{-1}{\underset{\shortparallel}{h_3}}\underset{2}{\underset{\shortparallel}{b_1}} + \underset{1}{\underset{\shortparallel}{h_4}}\underset{0}{\underset{\shortparallel}{b_0}} + \underset{0}{\underset{\shortparallel}{g_4}} = 8-4-2 = 2$