TMA683 Tillämpad matematik HT21

David Cohen

Chalmers University of Technology & University of Gothenburg

http://www.math.chalmers.se/~cohend/

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Chapter 1: Introduction and Motivation

Thanks to www.images.google.ch

Ordinary differential equations

Ordinary Differential Equations (ODE) often appear in the dynamical description of deterministic systems in physics, chemistry, biology, etc.

Ordinary Differential Equation = An equation that contains some derivatives of an unknown function (here, f and y_0 are given, y is unknown):

$$\begin{cases} \dot{y} = \frac{\mathrm{d}}{\mathrm{d}t} y(t) = f(y(t)) \\ y(0) = y_0. \end{cases}$$

Example of ODE: SIR model in epidemiology

Various models of epidemiology are described by ODE.

One popular example is the SIR model

$$\begin{aligned}
\frac{dS}{dt} &= \beta_S(S+I+R) - \mu_S S - \gamma_S SI \\
\frac{dI}{dt} &= \gamma_S SI - \mu_I I - \beta_R I \\
\frac{dR}{dt} &= \beta_R I - \mu_R R.
\end{aligned}$$

For fans of The Walking Dead, the above can also be used to model a zombie outbreak.

B. Calderhead, M. Girolami, D.J. Higham. Is It Safe To Go Out Yet? Statistical Inference in a Zombie Outbreak Model.

Partial differential equations

Partial Differential Equations (PDE) are more elaborate mathematical models used everywhere in science and engineering.

Partial Differential Equation = An equation that contains some partial derivatives of an unknown multivariable function.

Used in many areas of industrial applications (aeronotics (Gripen), cars (Volvo), etc.).

Example of a PDE: Fitzhugh–Nagumo system

A PDE model used to describe the propagation of nerve impulses read

$$\begin{cases} \frac{\partial}{\partial t} u(t,x) - \Delta u(t,x) = u(t,x)\left((1-u(t,x))(u(t,x)-\alpha) - v(t,x)\right) \\ \frac{\partial}{\partial t} v(t,x) = v_\infty(u(t,x)) - v(t,x), \end{cases}$$

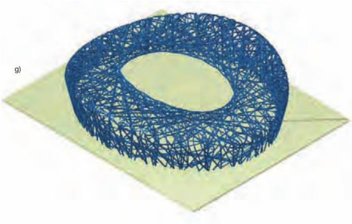
here u is called the membrane potential and v the recovery variable.

Spike=short, nonlinear elevation of membrane voltage u*, diminished over time by a slower, linear recovery variable* v *in a neuron after stimulation by an external input current.*

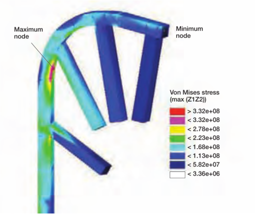
Content of the course TMA683

- Describe mathematical tools and spaces to study ODEs and PDEs.
- Present and analyse various computational techniques to approximate solutions to ODEs and PDEs.
- Provide theoretical tools to find exact solutions to particular ODEs and PDEs.

Applications: Structural analysis of a stadium



9. Finite element analysis at the elbow truss at the eave.



$3D$ structural analysis of a stadium.
Pictures taken from The Beijing National Stadium Special Issue $1/2009$, The Arup Journal.

Applications: Simulation of the crash of a train

click Crash of a train