Exempelgrafer för lösningar till Studio2, TMA683

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Differentialekvationen som ska lösas är

$$u_t(x,t) - u_{xx}(x,t) = f(x,t), \quad (x,t) \in (0,1) \times (0,T),$$

med randvillkor

$$u_x(0,t) = u(1,t) = 0$$

och begynnelsevillkor

$$u(x,0) = u_0(x).$$

I uppgift (a) sätter vi $f = 0$ och $u_0(x) = 1 - x$. Ekvationen kan då lösas med variabelseparering och vi får exakt lösning (se senare i kursen)

$$u_*(x,t) = \sum_{j=0}^{\infty} \frac{2}{\lambda_j^2} \exp(-\lambda_j^2 t) \cos(\lambda_j x),$$

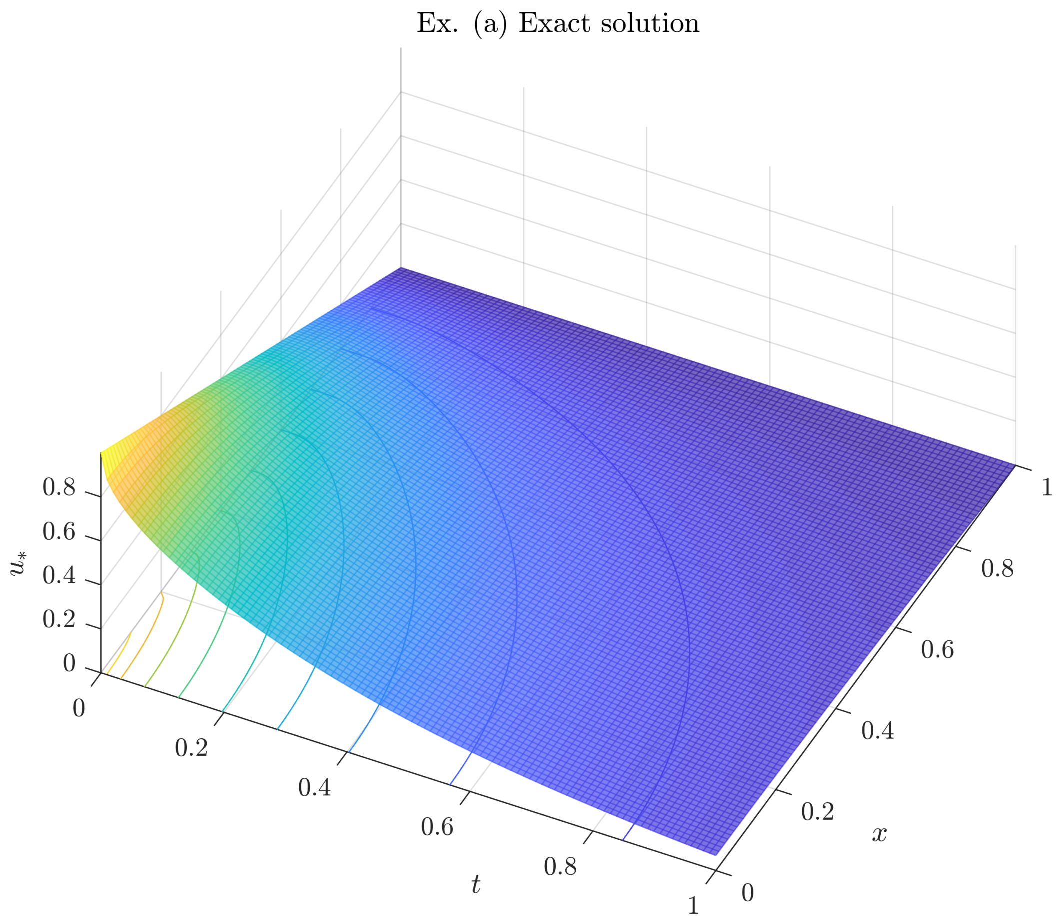
där $\lambda_j = \pi(j + 1/2)$.

Denna serie kan approximeras i Matlab genom att trunkera summan när termerna blir väldigt små.

Nedan följer kod för att jämföra FEM-lösningen med den "exakta" för uppgift (a), för olika upplösning på diskretiseringen.

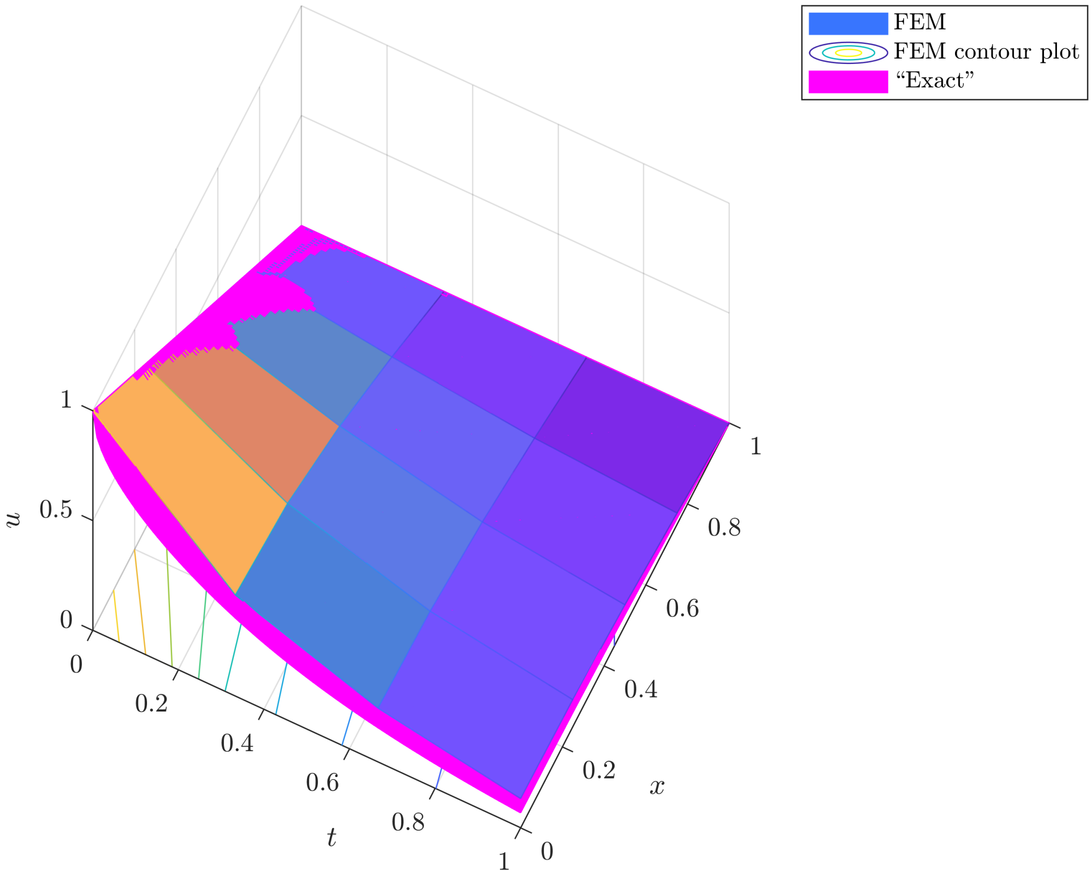
```
%% Exercise (a)

% params
T = 1;
x_0 = 0;
x_end = 1;

u0=@(x) 1-x; % initial value
f = @(x,t) 0*x; % right-hand-side
% (multiply by x above for compatibility when x is a vector)

% function file calculating the "reference" solution
```

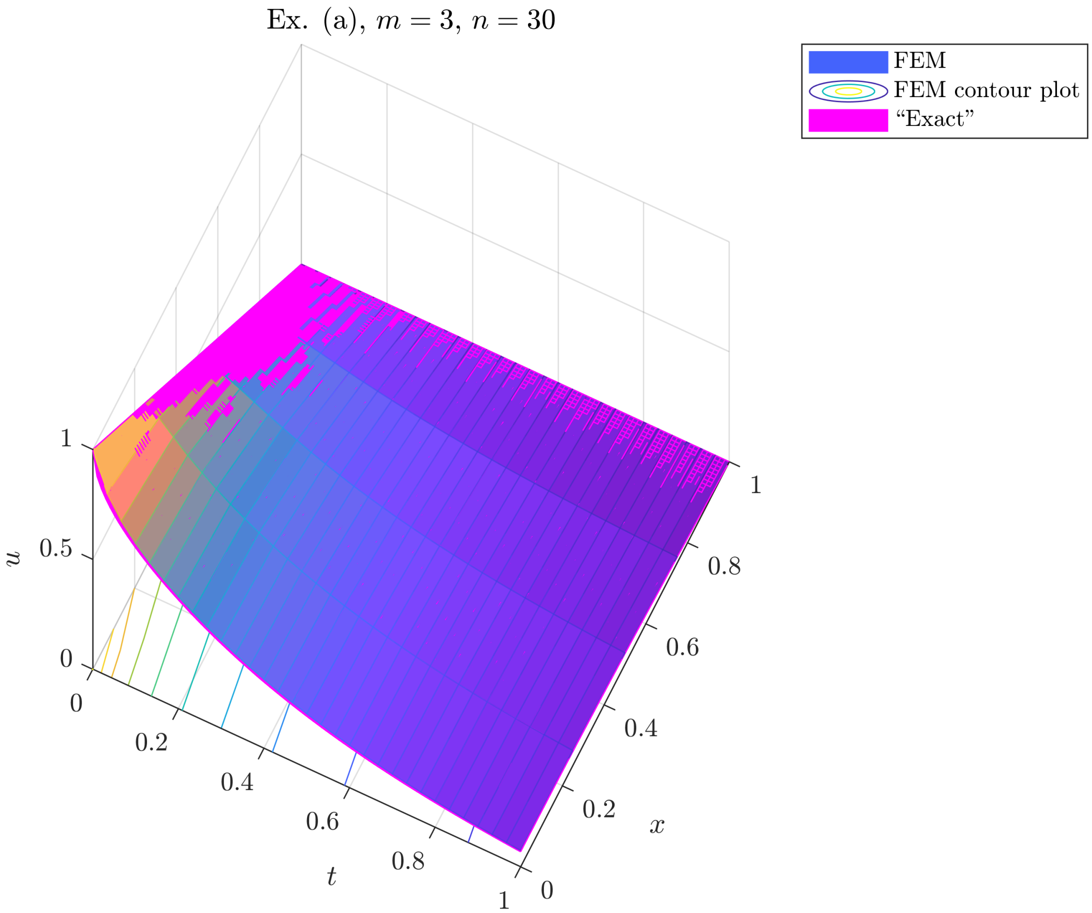
```
[t_v, x_v, exU] = Studio2_exact_sol_a(100, 100, T, x_0, x_end);
% settings only to get good latex export from matlab:
figure, set(gcf, 'Visible', 'on'), set(gcf, 'renderer', 'Painters')
% plot the reference solution:
% surfc plots both the surface and a contour plot (useful for ...
    checking the
% Neumann boundary condition)
surfc(t_v, x_v, exU, 'FaceAlpha', 0.7, 'EdgeAlpha', 0.7, ...
    'EdgeColor', 'interp')
title('Ex. (a) Reference solution')
% set viewing angle:
view([26 64])
% Note: to get correct tex-math mode (enclosed in $ below),
% you need to use the command
% set(0, 'defaultTextInterpreter', 'latex')
xlabel('$t$'); ylabel('$x$'); zlabel('$u_*$')
```



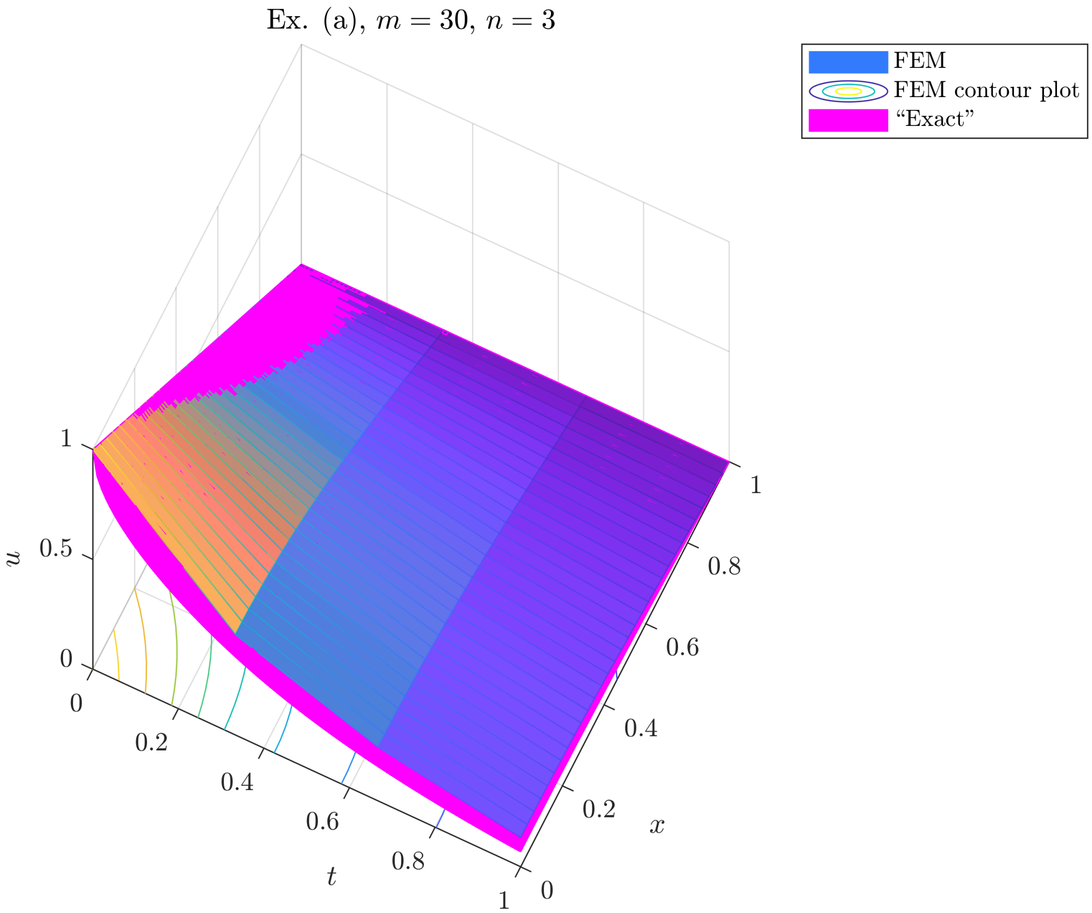
```matlab
% try different combinations of m and n
for m = [3, 30]
    for n = [3, 30]
        % function file calculating the FEM solution:
        [t_v2, x_v2, U] = Studio2_FEM_sol(m, n, f, u0, T, x_0, ...
            x_end);

        figure, set(gcf, 'Visible', 'on'), set(gcf, 'renderer', ...
            'Painters')
        surfc(t_v2, x_v2, U, 'EdgeColor', 'interp', 'FaceAlpha', ...
            0.7, 'EdgeAlpha', 0.7)
        hold on
        surf(t_v, x_v, exU, ...
             'EdgeColor', 'm', ...
             'FaceColor', 'm')
        view([26 64])
        title(sprintf('Ex. (a), $m=%d$, $n=%d$', m, n))
        xlabel('$t$'); ylabel('$x$'); zlabel('$u$')
        legend("FEM", "FEM contour plot", "Reference")
    end
end
```

Ex. (a), $m = 3$, $n = 3$

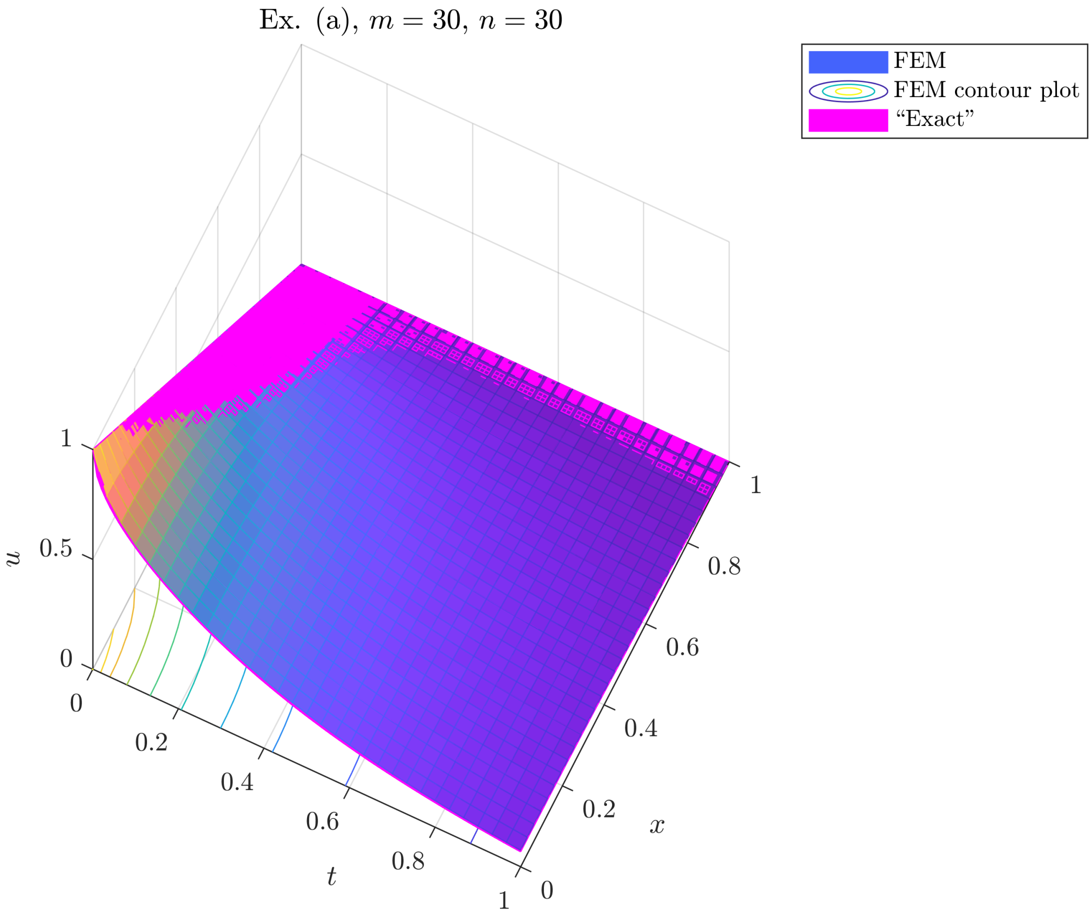




Ex. (a), $m = 3$, $n = 30$
FEM
FEM contour plot
"Exact"
u
1
0.5
0
0
0.2
0.4
0.6
0.8
1
t
0
0.2
0.4
0.6
0.8
1
x



Ex. (a), $m = 30$, $n = 3$
FEM
FEM contour plot
"Exact"
u
t
x
0
0.5
1
0.2
0.4
0.6
0.8



I (b)-uppgiften sätter vi $u_0(x) = \cos(\pi x/2)$ och

$$f(x,t) = \frac{10}{\sigma^2} \exp\left(-t - \frac{(\bar{x}-x)^2}{\sigma^2}\right),$$

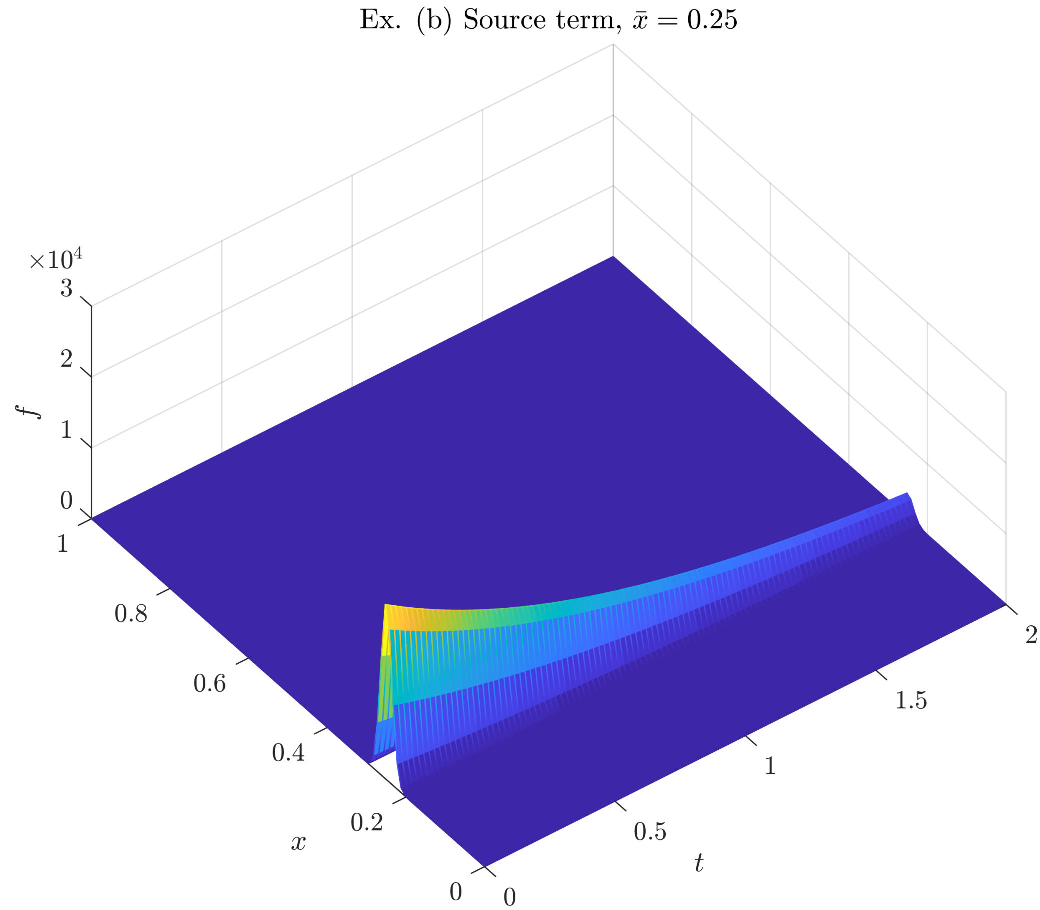
med $\sigma = 0.02$.

Här är det ännu svårare att hitta den exakta lösningen. Vi använder endast FEM nedan.

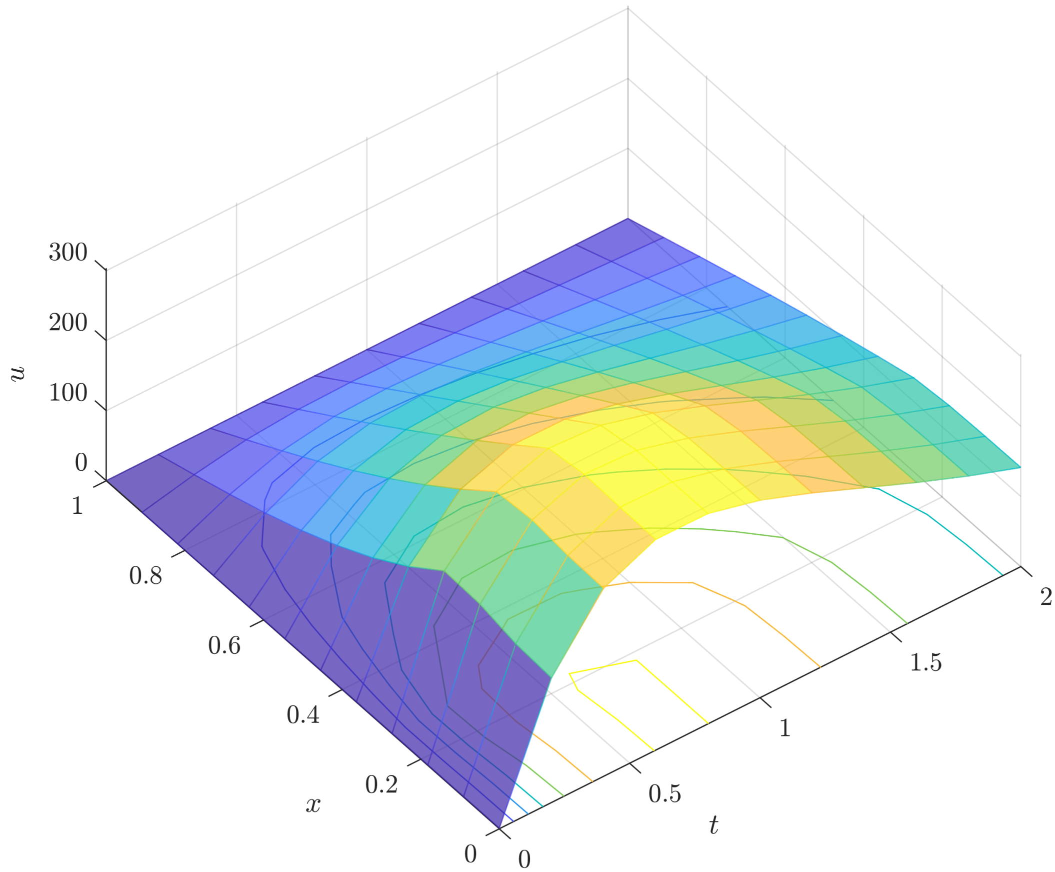
Nedan följer kod för att testa olika värden för antalet noder och $\bar{x}$.

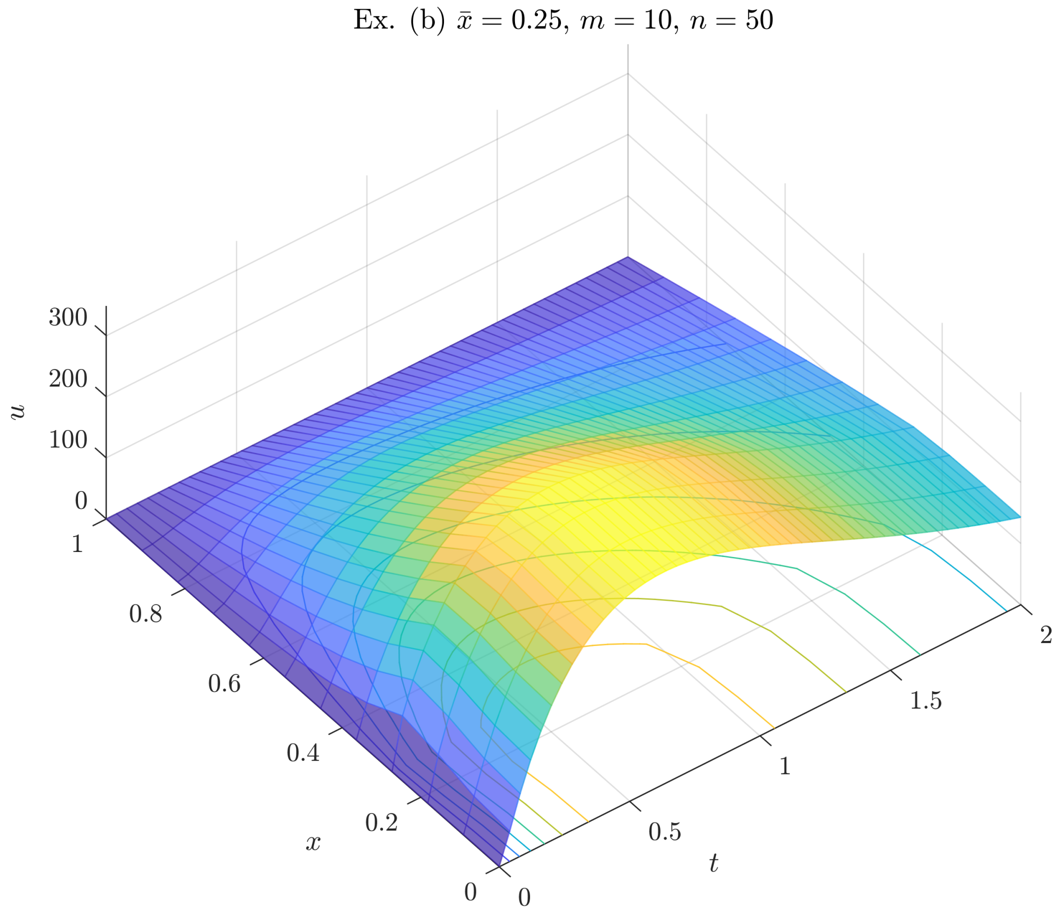
```
%% Exercise (b)

% params
T = 2;
```

```
x_0 = 0;
x_end = 1;

u0 = @(x) cos(pi*x/2); % initial value
sigma = 0.02;
f_m = @(x, t, xmean) 10/(sigma^2)*exp(-t-(x-xmean).^2/(sigma^2));


% try different combinations of xmean, m and n
for xmean = [1/4, 1/2, 3/4]
    f = @(x,t) f_m(x, t, xmean);
    % plot f
    figure, set(gcf, 'Visible', 'on'), set(gcf, 'renderer', ...
       'Painters')
    t_v = linspace(0,T);
    x_v = linspace(x_0, x_end);
    [T_M, X_M] = meshgrid(t_v, x_v);
    surf(t_v, x_v, f(X_M, T_M), 'EdgeColor', 'interp')
    view([-37 64])
    xlabel('$t$'); ylabel('$x$'); zlabel('$f$')
    title(sprintf('Ex. (b) Source term, $\\bar x = %g$', xmean))

    for m = [10, 50]
        for n = [10, 50]
            % function file calculating the FEM solution:
            [t_v, x_v, U] = Studio2_FEM_sol(m, n, f, u0, T, x_0, ...
               x_end);

            figure, set(gcf, 'Visible', 'on'), set(gcf, ...
               'renderer', 'Painters')
            surfc(t_v, x_v, U, 'EdgeColor', 'interp', ...
               'FaceAlpha', 0.7, 'EdgeAlpha', 0.7)
            view([-37 64])
            title(sprintf('Ex. (b) $\\bar{x}=%g$, $m=%d$, ...
               $n=%d$', xmean, m, n))
            xlabel('$t$'); ylabel('$x$'); zlabel('$u$')
        end
    end
end
```

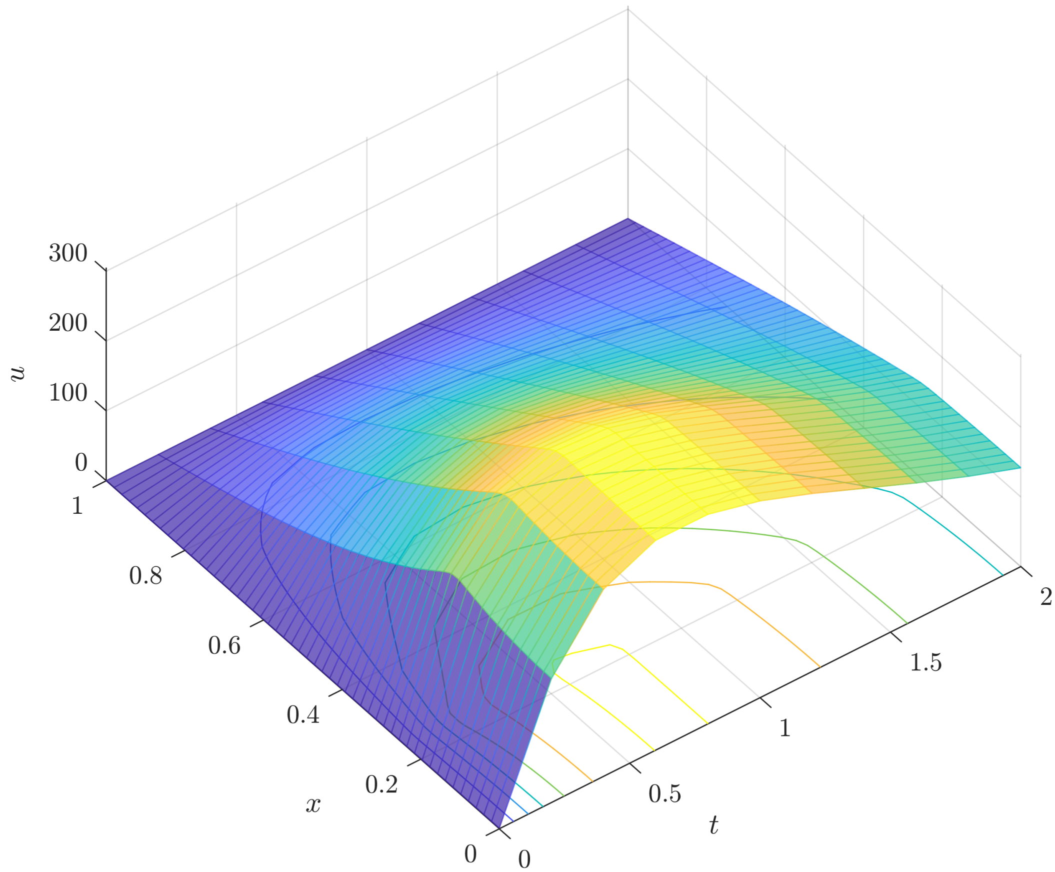

Ex. (b) Source term, $\bar{x} = 0.25$
$\times 10^4$
3
2
1
0
f
1
0.8
0.6
0.4
0.2
0
x
0
0.5
1
1.5
2
t

Ex. (b) $\bar{x} = 0.25$, $m = 10$, $n = 10$

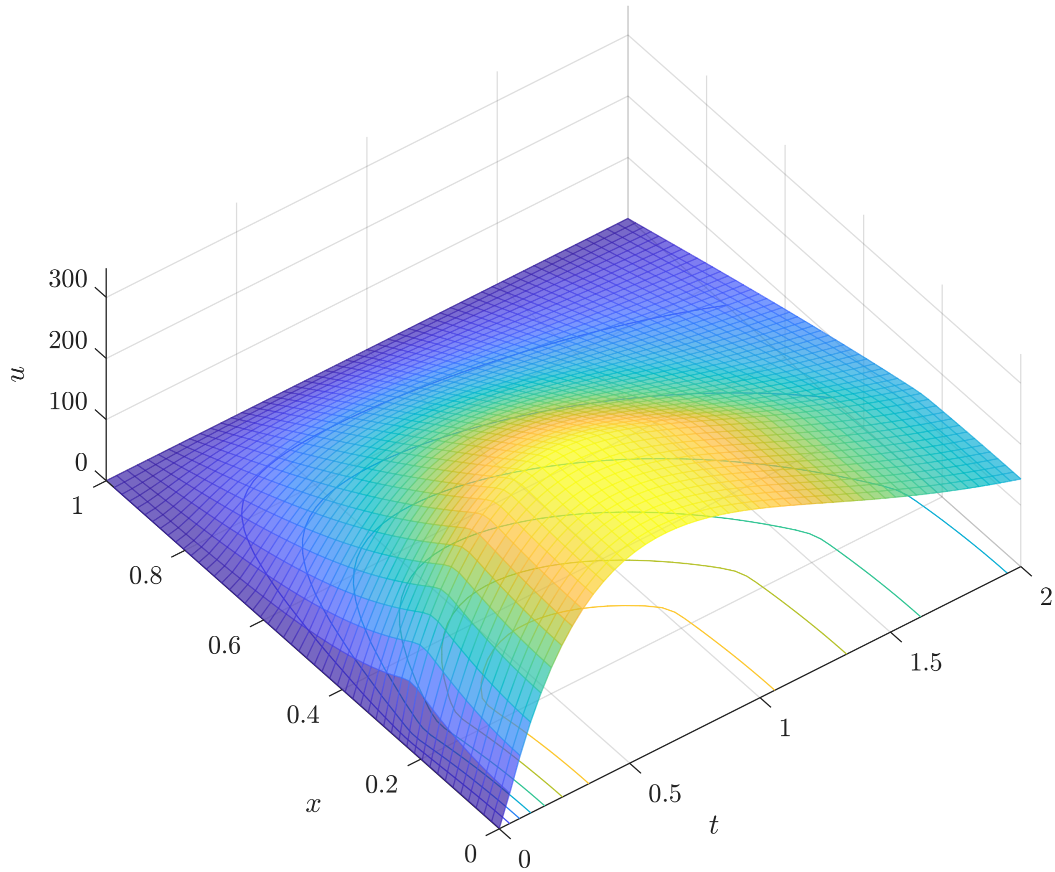



Ex. (b) $\bar{x} = 0.25$, $m = 10$, $n = 50$
300
200
100
0
u
1
0.8
0.6
0.4
0.2
0
x
0
0.5
1
1.5
2
t

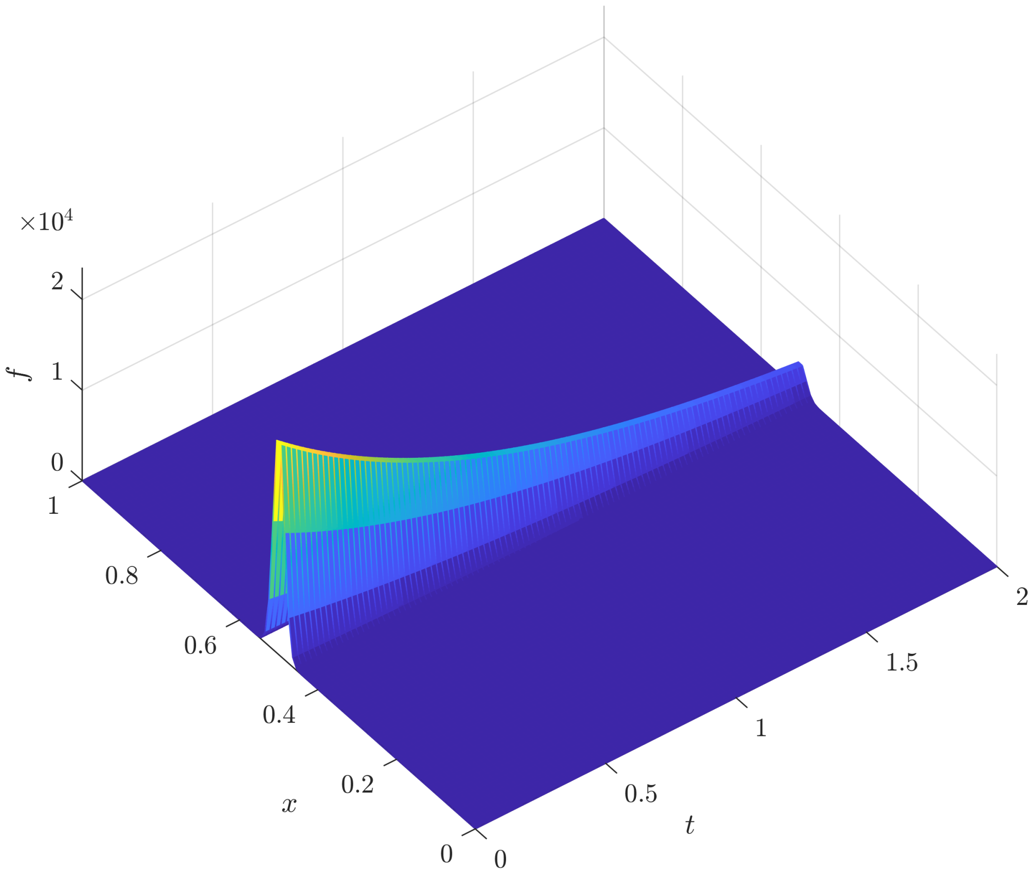
Ex. (b) $\bar{x} = 0.25$, $m = 50$, $n = 10$



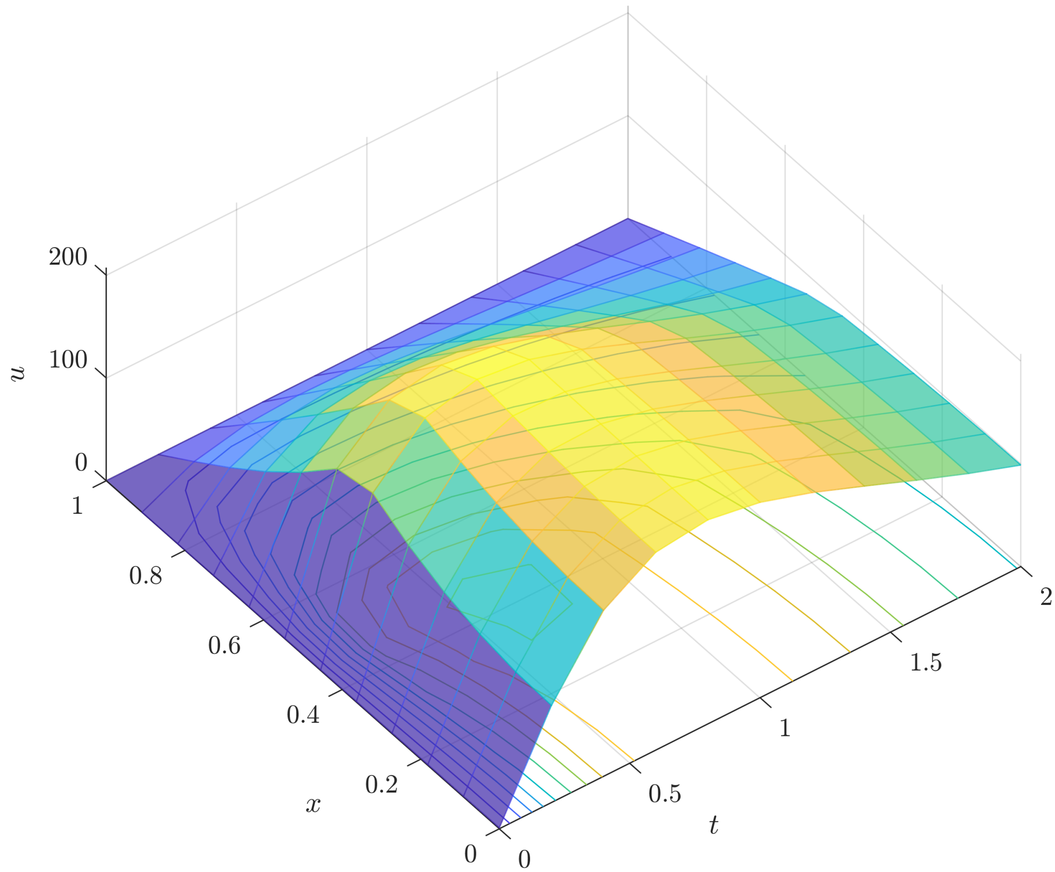
Ex. (b) $\bar{x} = 0.25$, $m = 50$, $n = 50$



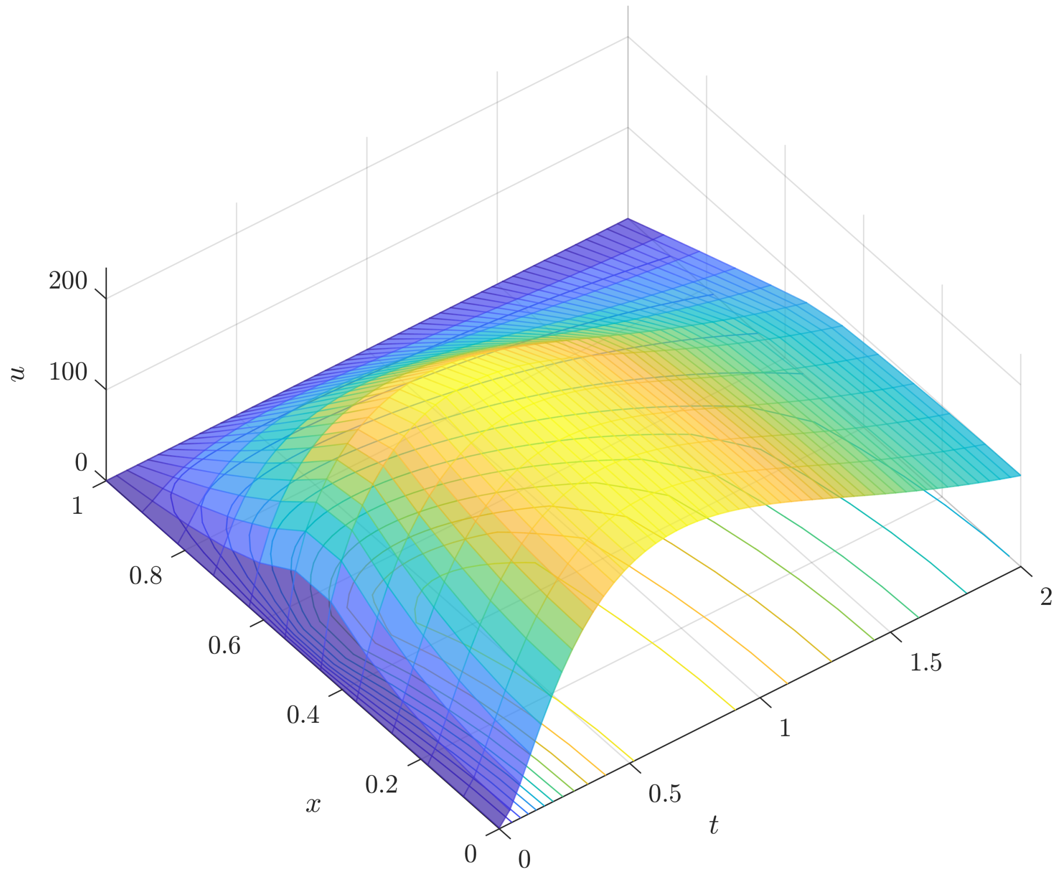
Ex. (b) Source term, $\bar{x} = 0.5$

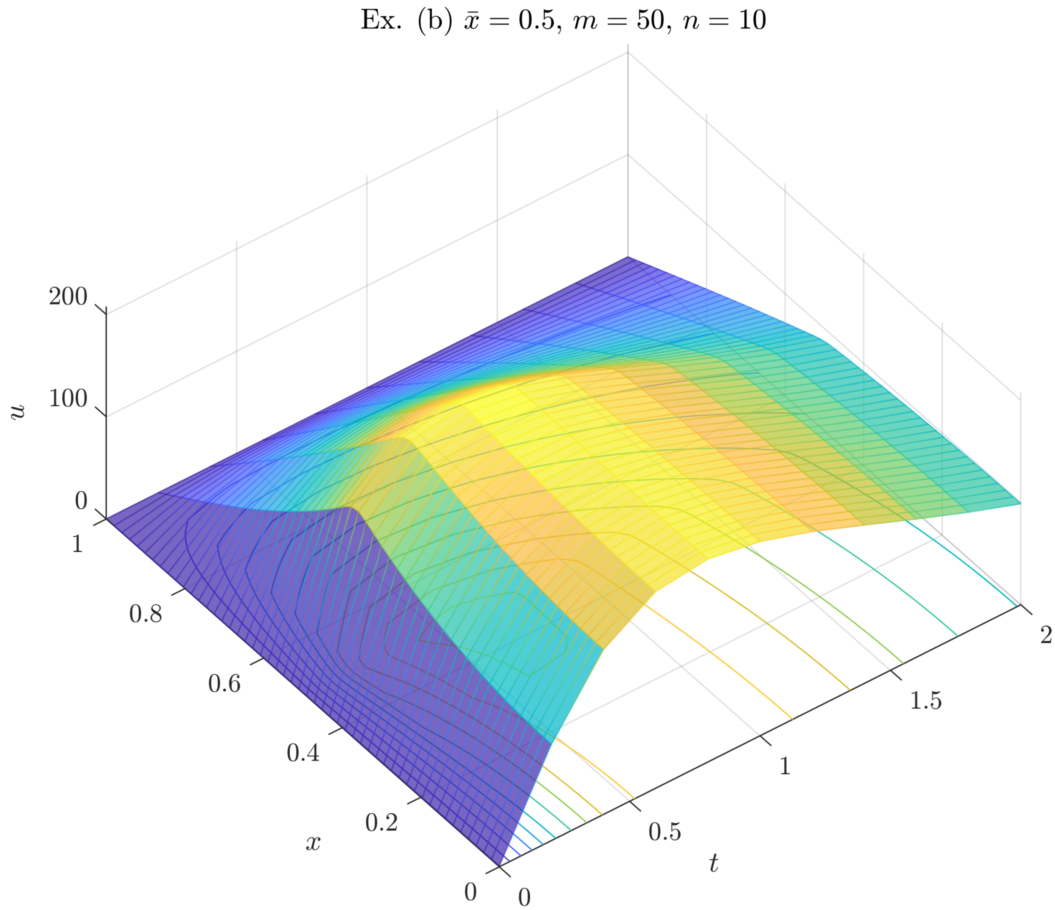


Ex. (b) $\bar{x} = 0.5$, $m = 10$, $n = 10$

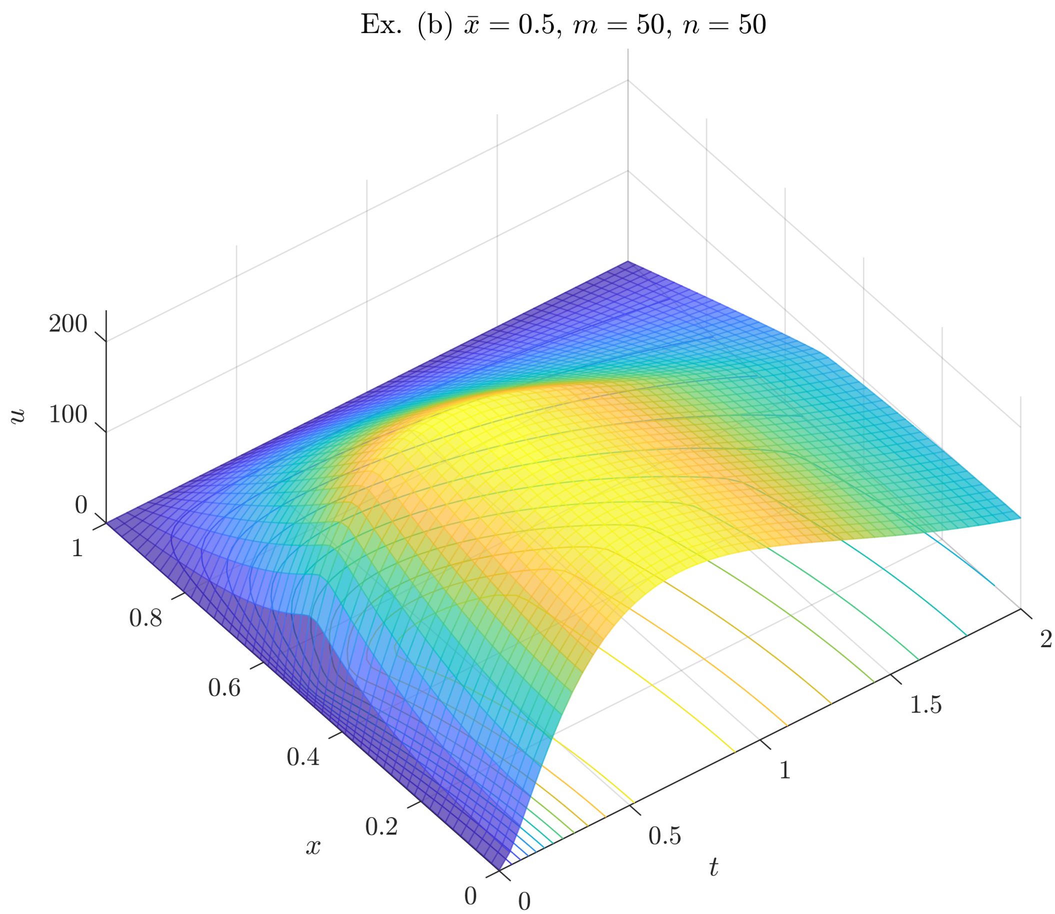


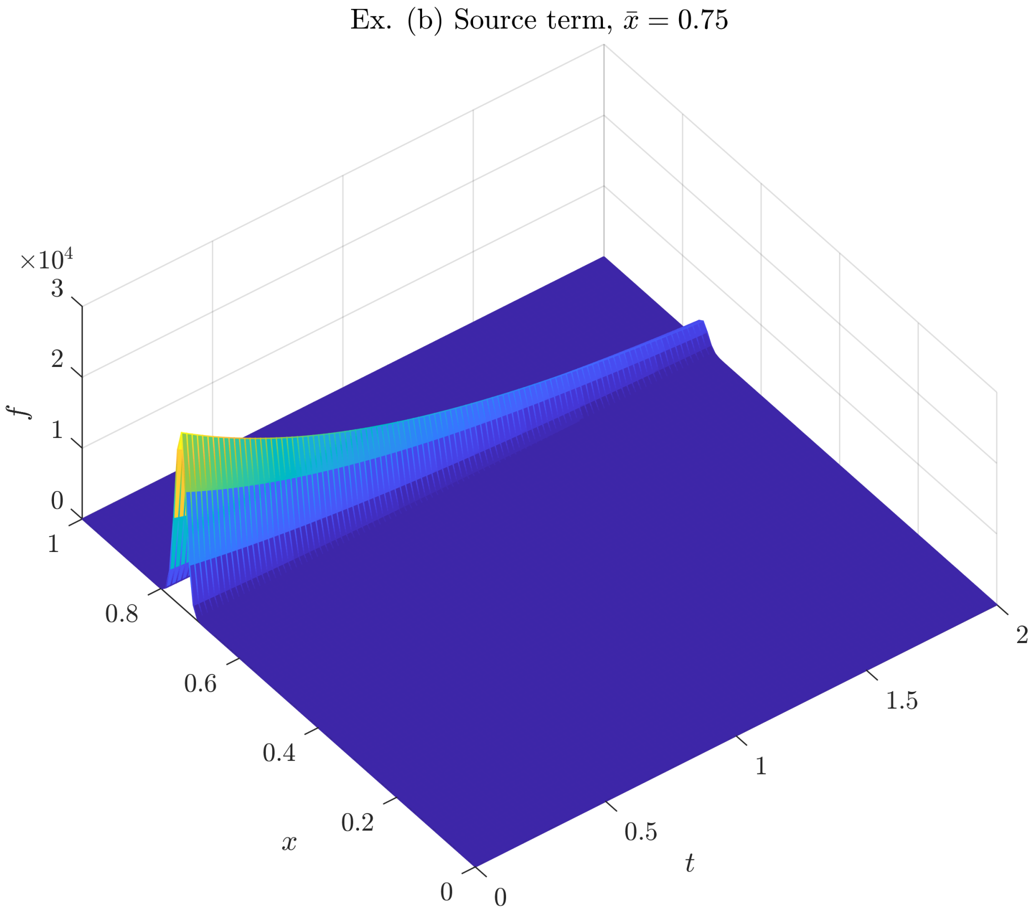
Ex. (b) $\bar{x} = 0.5$, $m = 10$, $n = 50$



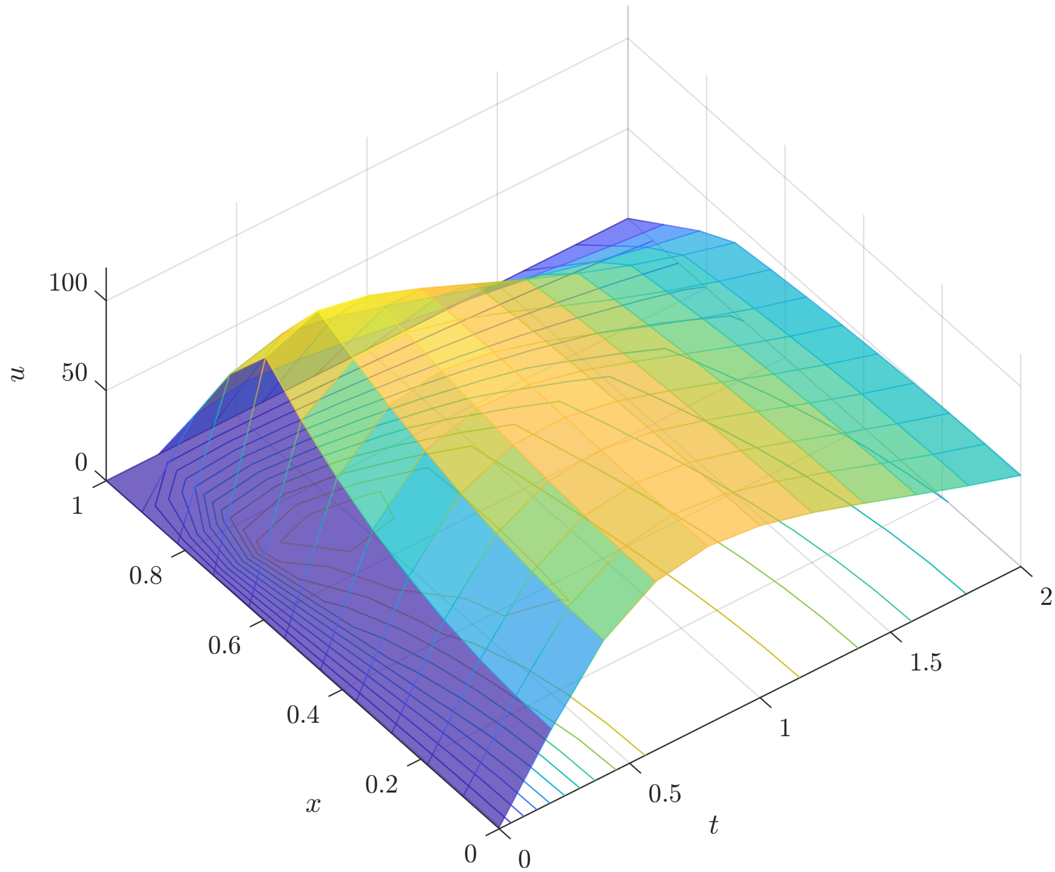


Ex. (b) $\bar{x} = 0.5$, $m = 50$, $n = 10$
200
100
0
u
1
0.8
0.6
0.4
0.2
0
x
0
0.5
1
1.5
2
t

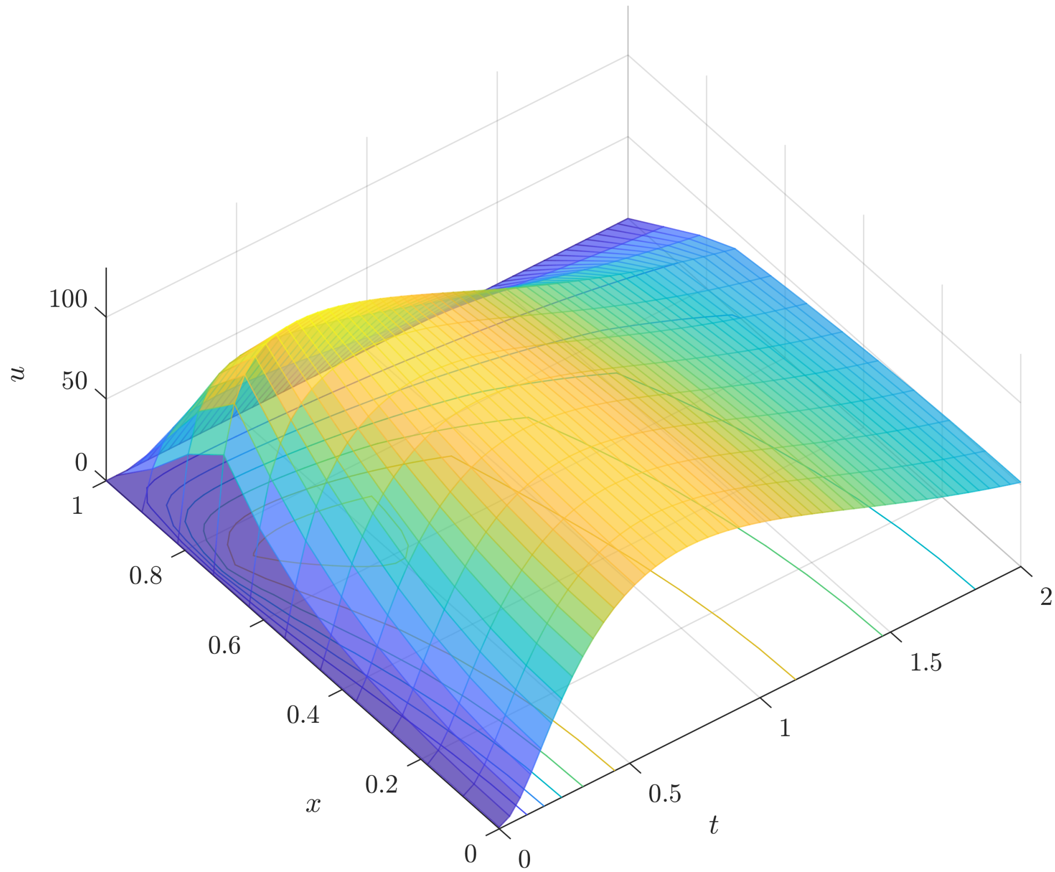

Ex. (b) $\bar{x} = 0.5$, $m = 50$, $n = 50$
u
200
100
0
1
0.8
0.6
0.4
0.2
0
x
0
0.5
1
1.5
2
t


Ex. (b) Source term, $\bar{x} = 0.75$
$\times 10^4$
3
2
1
0
f
1
0.8
0.6
0.4
0.2
0
x
0
0.5
1
1.5
2
t

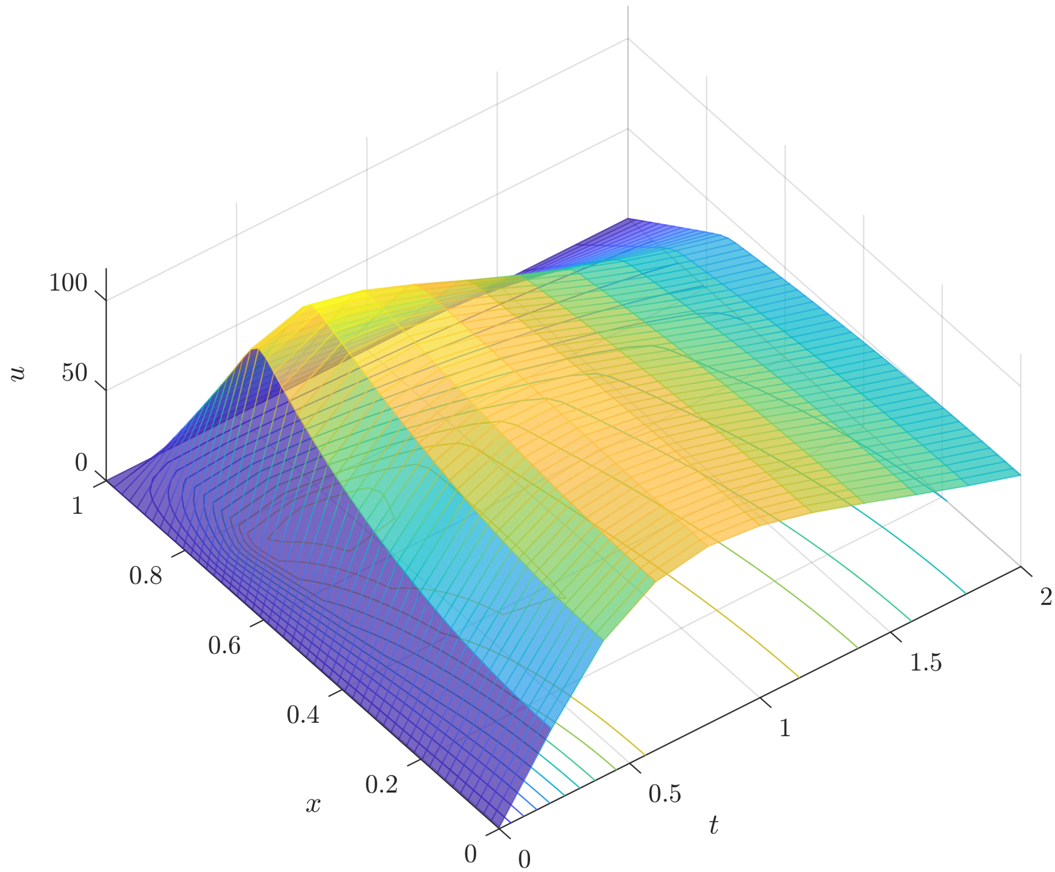
Ex. (b) $\bar{x} = 0.75$, $m = 10$, $n = 10$



Ex. (b) $\bar{x} = 0.75$, $m = 10$, $n = 50$



Ex. (b) $\bar{x} = 0.75$, $m = 50$, $n = 10$



Ex. (b) $\bar{x} = 0.75$, $m = 50$, $n = 50$