Optimization in the aviation industry

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Agenda

- Introduction
  - Jeppesen/Boeing
  - The aviation business
  - Our products
- The rostering problem
  - Modeling
  - Challenges
  - Solution
- Work opportunities

Who I am

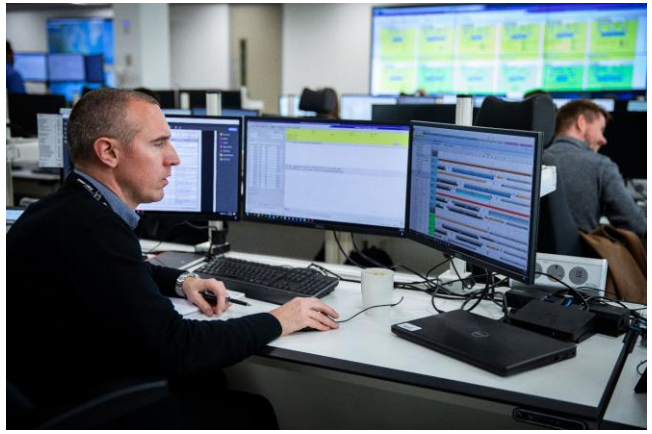
Emily Curry

- Attended Engineering Mathematics at Chalmers 2013-2018
- Master’s in Engineering Mathematics and Computational Science

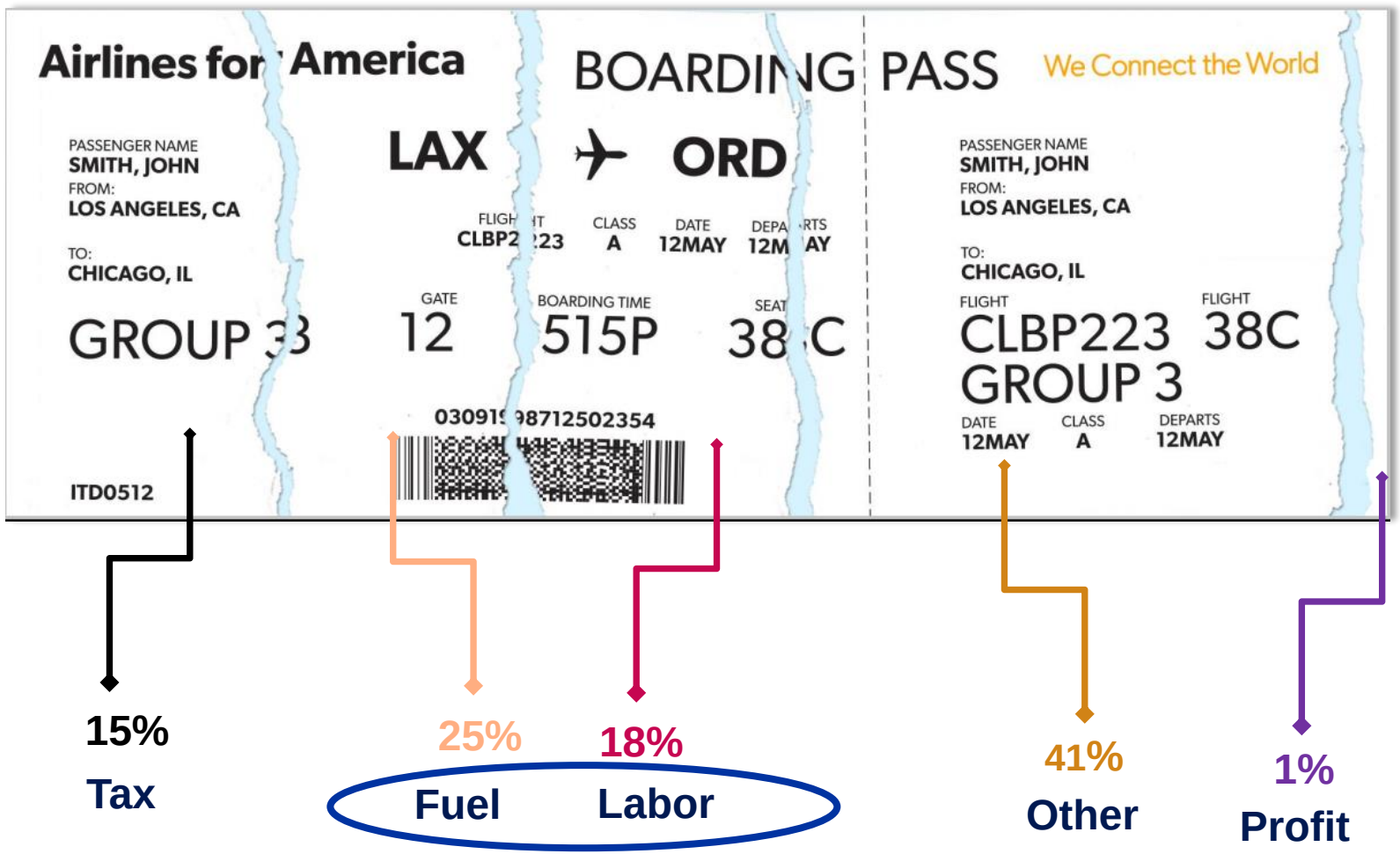
- At Boeing:
  - Optimization Expert ~3 years
  - Product Owner ~6 months
  - Participant in 2021 Emerging Talent in Europe Council (ETEC)

Who we are

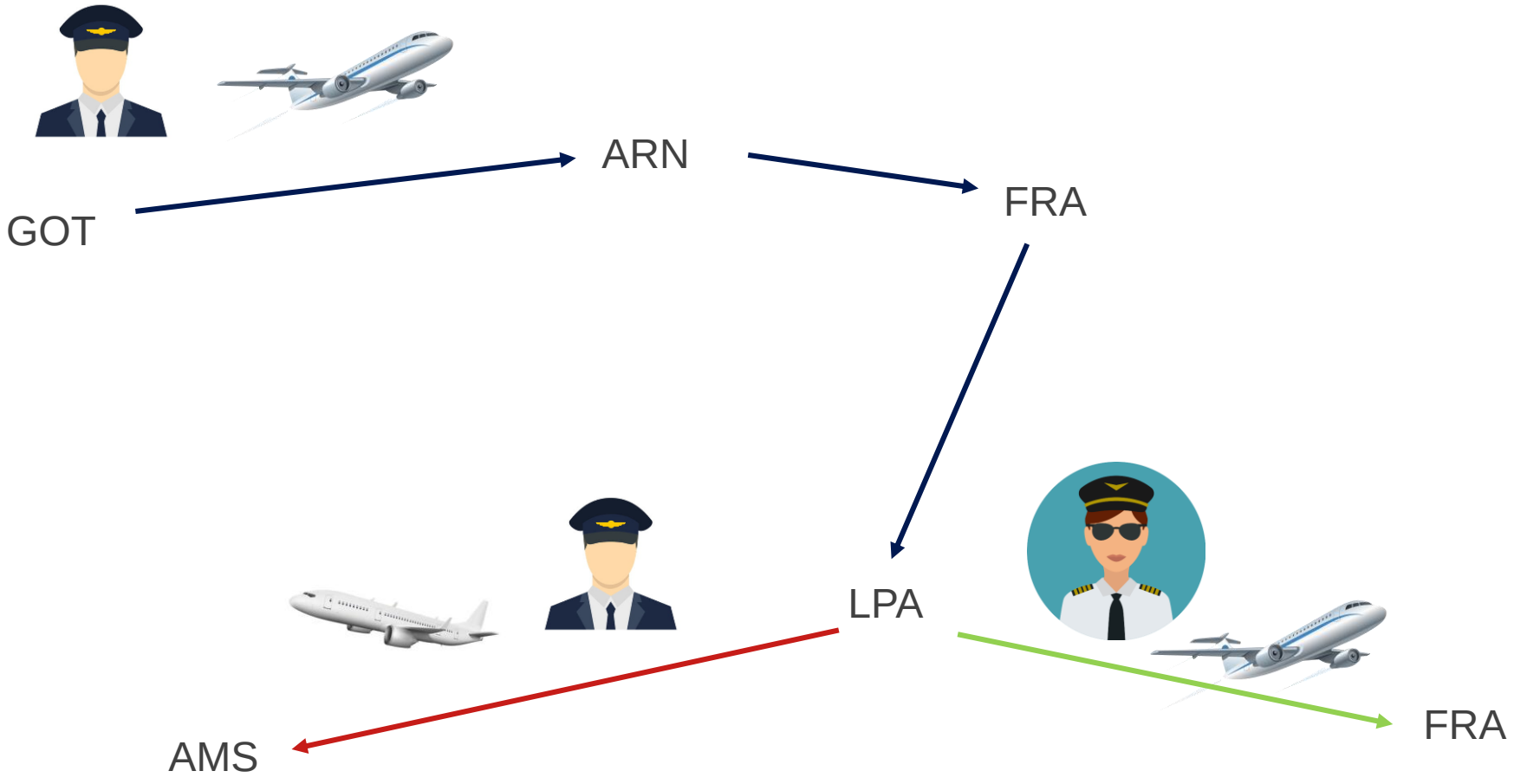
- History:
  - Started as a Volvo project
  - SAS interested
  - Carmen Systems founded in 1994
  - Acquired by Jeppesen 2006
  - Boeing integration
- The office:
  - ~300 employees
  - ~35 different nationalities
- Focus on airline planning for crew and fleet



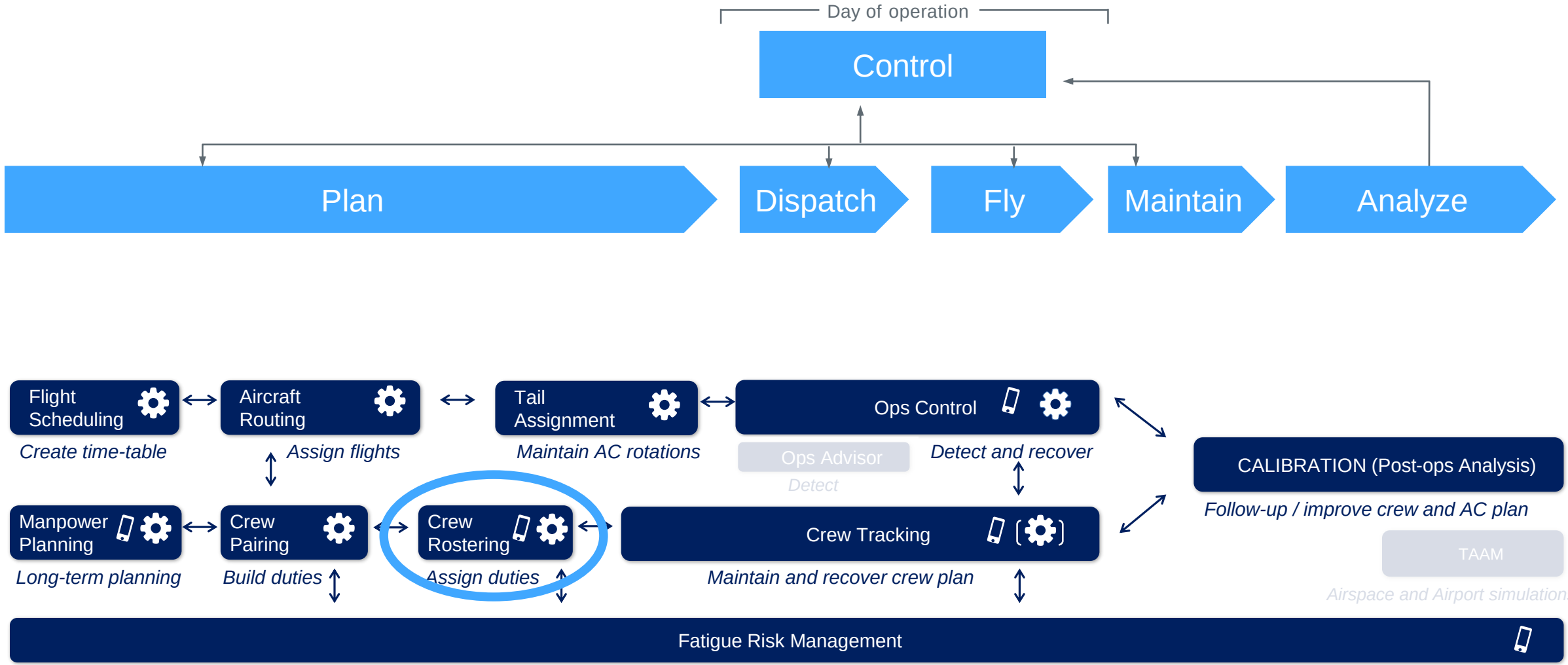
Why use our products



Planning



Our products

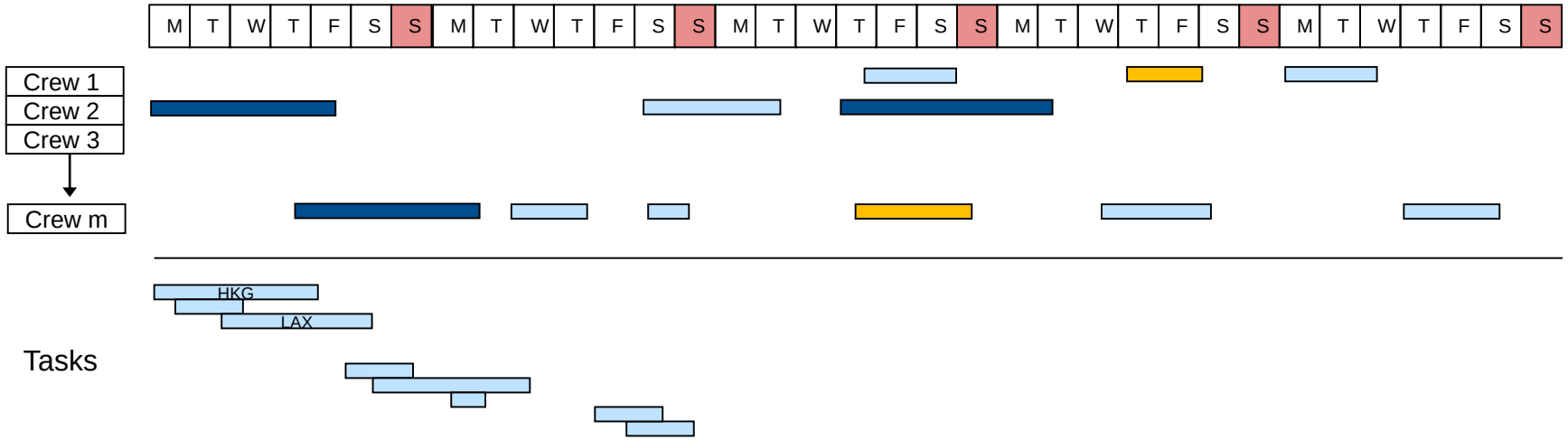


Current Rostering Customers (as of June 9)

The rostering problem

(roster = personal, monthly schedule)

Want to assign all task, such that ”cost” is minimized and legal/contractual rules are respected:



What is optimization?

Minimize

objective function

Subject to

Constraints

Example:

Volume of a soda can

Objective: Use as little material as possible

Minimize area

Subject to volume = V

minimize $2\pi r^2 + 2\pi rh$

subject to: $\pi r^2 h = V$

$r \geq 0, h \geq 0$

The rostering optimization problem

Objective: Minimize cost

Constraints: Make sure that all trips are assigned to crew and legal/contractual rules are satisfied

Min ”cost”

Subject to

all crew assigned to exactly one roster

all trips assigned to crew

all rules satisfied

In mathematical form

$$\text{minimize } \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}$$

Objective function

$$\text{subject to } A\boldsymbol{x} = \mathbf{1}$$

Coverage constraints
- Each crew has 1 roster
- Each trip is assigned

$$\boldsymbol{x} \in \{0,1\}^n$$

$$\text{where } x_i = \begin{cases} 1 \text{ if roster } i \text{ is assigned} \\ 0 \text{ else} \end{cases} \quad \text{for } i = 1, 2, .., n$$

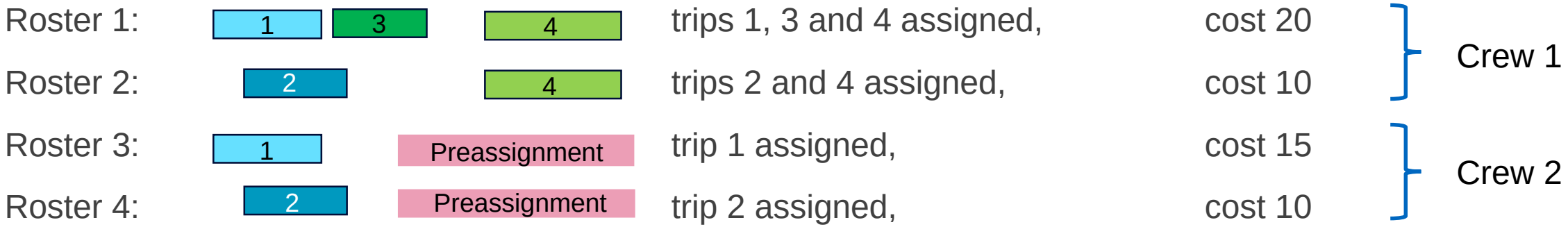
Assignable to crew 1

$$A = [\boldsymbol{a}_1 \quad \boldsymbol{a}_2 \quad \dots \quad \boldsymbol{a}_n], \text{where e.g. } \boldsymbol{a}_j = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 1 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \quad j \in \{1, 2, \dots n\}$$

Number of crew

Number of trips

Small example: 2 crew, 4 trips, 4 rosters



Optimization

$$\min c^T x = \min c_1 x_1 + c_2 x_2 + c_3 x_3 + c_4 x_4 = \min 20\, x_1 + 10 x_2 + 15 x_3 + 10 x_4$$

Crew 1
Trip 1
Trip 3
Trip 4

$$\text{subject to } Ax = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_3 + x_4 \\ x_1 + x_3 \\ x_2 + x_4 \\ x_1 \\ x_1 + x_2 \end{bmatrix} \geq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$x \in \{0,1\}^n$$

Crew 1 should get either roster 1 or 2

Trip 1 should be part of roster 1 or 3

Optimal solution:

$x_1 = 1, x_2 = 0, x_3 = 0, x_4 = 1$

Assign Roster 1 (to crew 1)
and Roster 4 (to crew 2)

Total cost: 30

In reality: Combinatorial explosion

Small problem: 10 crew, 50 trips

- Possible rosters for 1 crew:
- *C(50, 5)* ≈ 2 000 000
- Possible solutions:
- *C(#possible rosters, 10)* ≈ 10^{60}

Largest test case: 22 000 crew, 190 000 trips

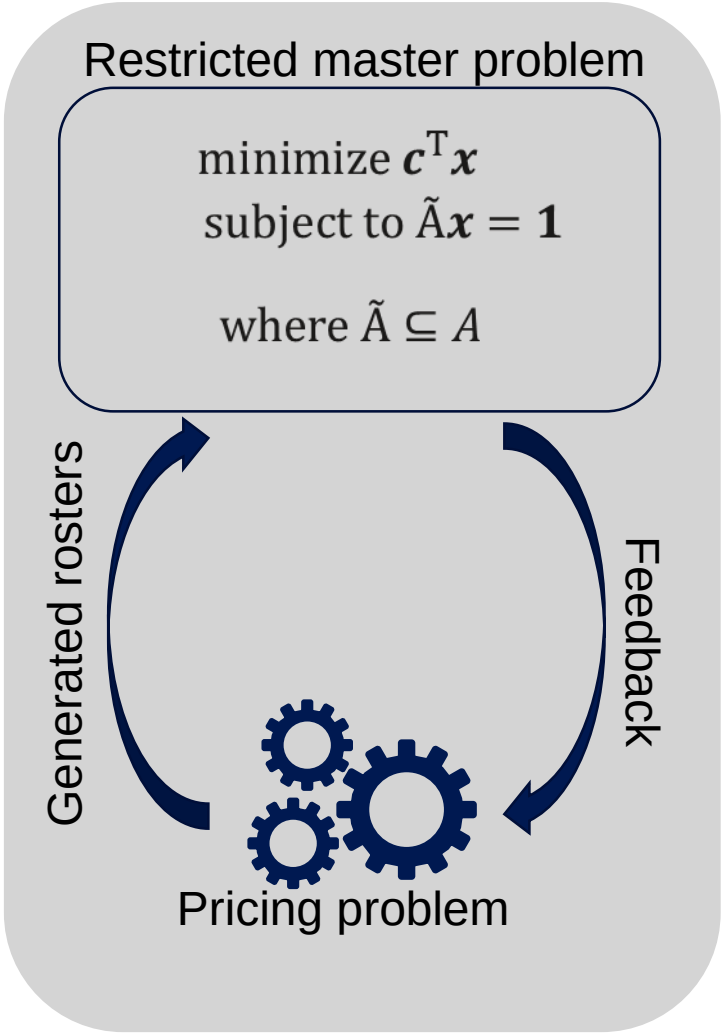
- → ~$10^{500\,000}$ possible solutions!
- (Assuming any assignment is legal)

~10^{18} grains of sand on earth!

The solution

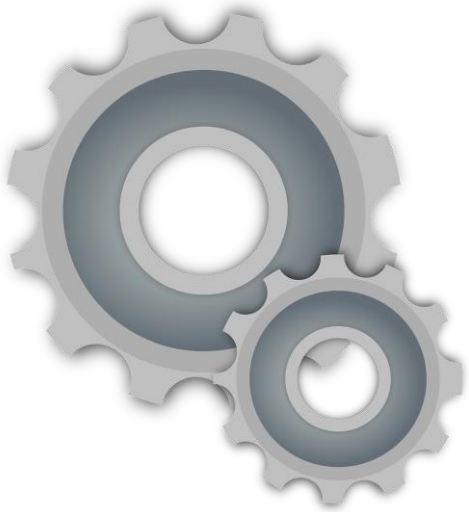
Column Generation

1. Set up smaller problem
2. Solve smaller problem
3. Try to generate new columns (rosters)
4. Repeat steps 2 and 3 until no more rosters can be generated



Efficient optimization

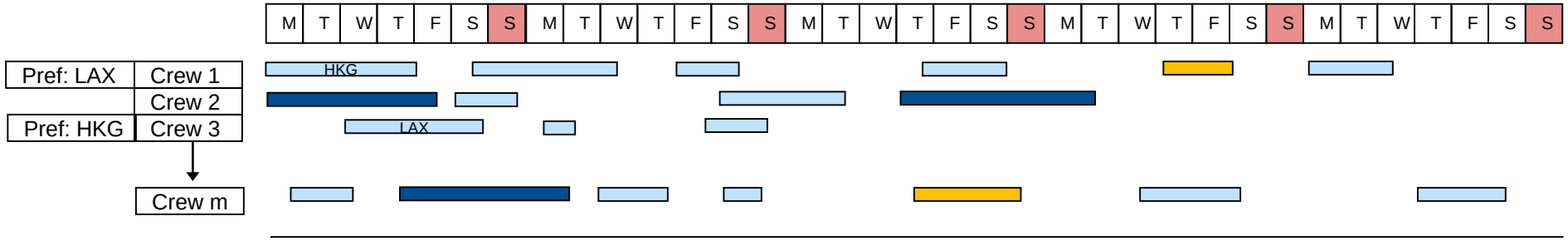
- Use column generation framework to solve the problem
- Rely on heuristics to generate solution within reasonable time



”We cannot solve these problems optimally within reasonable time... But as long as we are the best in the world at it, it doesn’t matter”

What is the best solution?

Cheapest solution = Best solution?



Other aspects:

- Satisfaction
- Fairness
- Robustness

Covid: Current situation

- Late changes to schedules
  - Travel restrictions
  - Crew sick
- Our products help customers quickly adapt
  - Rewrite and add new rules
  - Replan
- Focus on supporting our customers
  - Offer business consulting services
  - Quickly adapt to changes
  - Different modeling

Opportunities

- Thesis work:
- Contact thesis@jeppesen.com if interested

- Work opportunities:
- Data Scientist, Software application developer, business consultant, service manager etc.
- Skills: Problem solving, algorithms, optimization, programming
- See jobs.boeing.com

Talk to us at FARM November 18

- Starting at Boeing: Academy and courses
- Products, Infrastructure, Programming languages
- Course participants are often a mix of customers and internal employees

Summary

- Why do we want to solve this problem?

- Right problem
- Right(?) solution
- Efficient solution

Thank you for listening

Questions?


BOEING