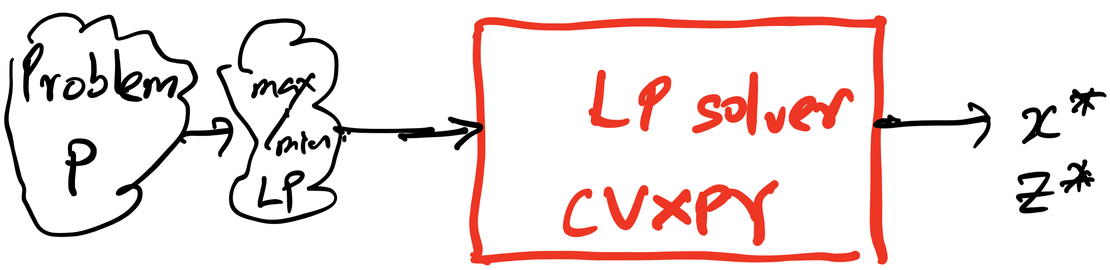
Lecture 2: Using LP

Recap: LP is

- max/min linear function
- real variables
- subject to linear constraints



Problem 1 (Diet Problem)

Input data:

	Carrots	Cabbage	Cucumber	Reqd.

Vita. A (mg/kg)	35	0.5	0.5	0.5
vita. B mg/kg	60	300	10	15
Fiber g/kg	30	20	10	4
Price £/kg.	0.75	0.5	0.15	

Problem: How to compose a diet that meets all nutrition requirements & minimize cost.

Modelling with variables.

$x_1 \geq 0$ kg of carrots

$x_2 \geq 0$ kg of cabbage

$x_3 \geq 0$ kg of cucumber

min $\quad 0.5\, x_1 + 0.75\, x_2 + 0.15\, x_3$

s.t. $\quad 35\, x_1 + 0.5\, x_2 + 0.5\, x_3 \geq 0.5$

(vit. A)

$$60x_1 + 300x_2 + 10x_3 \geq 15$$

(vit. B)

$$30x_1 + 20x_2 + 10x_3 \geq 4$$

(fiber)

$$x_1, x_2, x_3 \geq 0$$

First problems solved using LP (1947)

77 variables

9 constraints

200 man years! with hand held calculators.

L. Kantarovich (1940...)
USSR

T. Koopmans (1960...)

economist

G. Dantzig (1960-70)

<u>Problem 2.</u> (Routing Data)

- $G = (V, E)$

  vertices: routers

  special vertex 0 source

  n dest.

- $c : E \to \mathbb{R}^{\geq 0}$

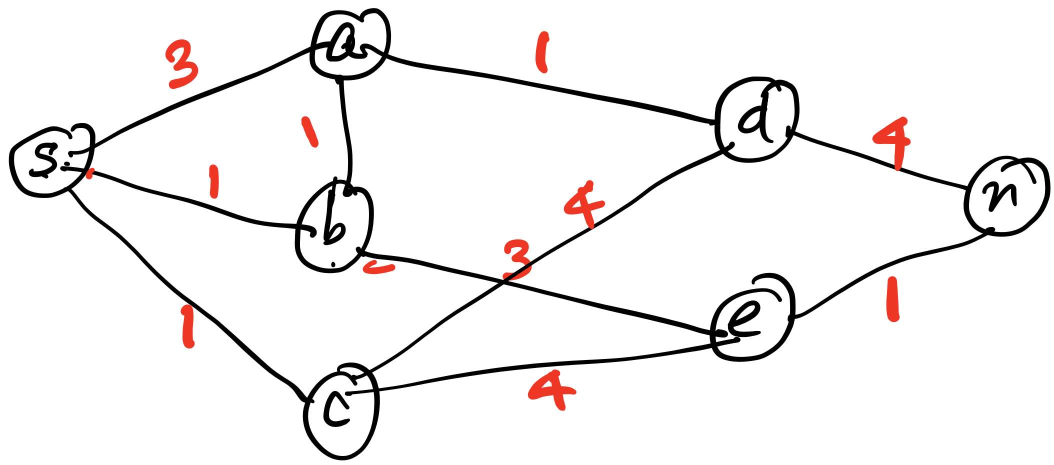
  max. data comm. limit.

Problem: stream data from source to dest.

at max. rate. sub. to constraints

i.e. using available links

& their max capacities.



variables

x_{sa}, x_{sb}, x_{sc}

x_{ab}, x_{ad},

x_{be}

x_{cd}, x_{ce}

x_{dn} x_{en}

interpretion:

x_{ij} amount of data (mb/s) sent from i to j

$$\max \quad x_{sa} + x_{sb} + x_{sc} =: z$$

s.t.

$$x_{sa} = x_{ab} + x_{ad}$$

(Conservation a)

$$x_{sb} + x_{ab} = x_{be}$$

$$x_{sc} = x_{cd} + x_{ce}$$

$$x_{ad} + x_{cd} = x_{dn}$$

$$x_{be} + x_{ce} = x_{en}$$

$$-3 \le x_{sa} \le 3$$

$$-1 \le x_{sb} \le 1$$

$$-1 \le x_{sc} \le 1$$

$$\vdots$$

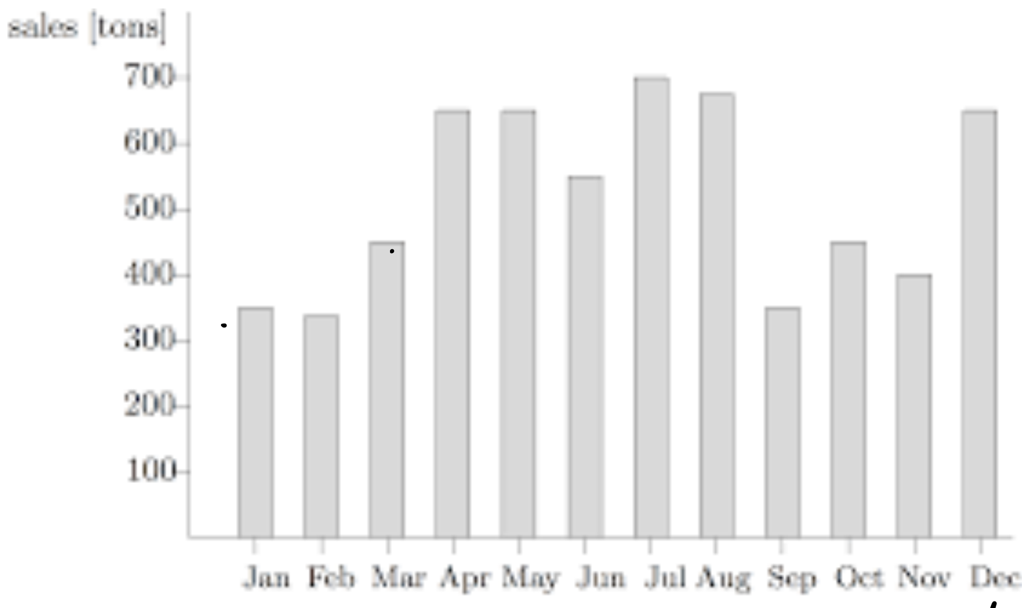
$x^*_{sa} = 2$

$\vdots$

$x^*_{ce} = -1$

$z^* = 4$

Problem 3 (Ice Cream)



$d_1 \; d_2 \qquad \qquad d_{12}$

- don't want to have drastic changes in production from one month to next
  ≤ 50 ton change.
- allowed to store some ice cream to be

used in later months.

Storage cost: $\$20$ / ton.

- How much to produce each month?
- How much to store each month?
- Minimize total cost!

Variables

$$x_1, x_2, \ldots, x_{12} \geq 0$$

x_i = amount produced in month i

$$s_1, s_2, \ldots, s_{12} \geq 0$$

s_i = amount stored in month i

min. $$\sum_{i=1}^{12} 20\, s_i + \sum_{i=1}^{12} 50 \underbrace{\left| x_i - x_{i-1} \right|}_{z_i}$$

s.t.

(meet demand in month i)

$$x_i + s_{i-1} \geqslant d_i \qquad i=1,\dots 12$$

$$x_i + s_{i-1} - s_i = d_i \qquad i=1,\dots,12$$

$$s_0 = 0 \qquad s_{12} = 0$$

$$x_1, \dots, x_{12} \geqslant 0, \quad s_1, \dots, s_{12} \geqslant 0$$

$$z_i \geqslant x_i - x_{i-1}$$

$$z_i \geqslant x_{i-1} - x_i$$

$$z_i \geqslant 0 \qquad i=1,\dots,12$$