Hints on Exercises

4.5

i) Use definition of conditional expectation and apply towering property with a filtration $\mathcal{F}_t$.
ii) In (a), apply Feynman-Kac. In (b), apply Itô's formula on $v_t u$ to get

$$\mathrm{d}(v_t(s,X_s)u(s,X_s)) = v_t(s,X_s)\mathrm{d}(u(s,X_s)) + u(s,X_s)\mathrm{d}(v_t(s,X_s)) + \mathrm{d}\langle v_t(s,X_s), u(s,X_s)\rangle.$$

To get an expression for $\mathrm{d}v_t$, one may apply Itô's formula on $f(\overline{X}_t)$ and take the expectation. To deal with the first term, one can check if u is a martingale and think about what happens to a stochastic integral with respect to a martingale. For (c), one may take the expectation of a continuous bounded function f, and see if the two are equal.

iii) Note that $X_T^{t,x}$ is Gaussian and use the cdf of a Gaussian distribution. Also,

$$\mathbb{P}(Y \in (a-r, a+r)) = \mathbb{P}(Y \leq a+r) - \mathbb{P}(Y \leq a-r).$$

5.2

1. Apply Itô's formula on $b(x)$ and $\sigma(x)$ and integrate with respect to s and W_s respectively. When taking the expectation of the squared expression, which integral/-s is/are sufficient to analyze? (Some decay faster than others).

5.3

1. Compute X_T by applying Itô's formula on some convenient function, and proceed by computing $\mathbb{E}(X_T^2)$.

2. Write

$$\mathbb{E}\big(\big(X_{(i+1)h}^{(h)}\big)^2\big) = \mathbb{E}\big(\big((X_{(i+1)h}^{(h)} - X_{ih}^{(h)}) + X_{ih}^{(h)}\big)^2\big).$$

Expand the square and write every expression in terms of y_i by using the definition of the Euler scheme.