

ORAL EXAMINATION, JUNE 2020

Name:

Date:

In this examination you are asked to state and prove the following result:

- The upcrossings inequality for submartingales (‘Snell’s upcrossing inequality’), Theorem 12.3.3 in Grimmett–Stirzaker

as well as your **choice of two** of the following:

- (1) Kolmogorov’s 0/1 law (Theorem 7.3.15)
- (2) The Renewal-Reward theorem (Theorem 10.5.1)
- (3) The backward martingale convergence theorem (Theorem 12.7.4)
- (4) The Paley–Wiener–Zygmund theorem about nowhere Lipschitz continuity of Brownian Motion (Theorem 1.30 in Mörters–Peres)

THE UPCROSSINGS INEQUALITY

Let $a < b$ and

$$\begin{aligned} S_1 &= \min\{n \geq 0 : Y_n \leq a\}, & T_1 &= \min\{n > S_1 : Y_n \geq b\}, \\ S_k &= \min\{n > T_{k-1} : Y_n \leq a\}, & T_k &= \min\{n > S_k : Y_n \geq b\}, \\ U_n(a, b; Y) &= \max\{k : T_k \leq n\} \end{aligned}$$

Statement: