

PROOFS IN THE COURSE

Here is a list of the theorems whose proofs I may ask you to explain on the final exam (written or oral). I may also ask you to define any of the terms or concepts which are relevant to the results on this list.

Theorems from Grimmett & Stirzaker:

- Theorem 7.3.10 (Borel–Cantelli Lemmas)
- Theorem 7.3.15 (Kolmogorov’s 0/1 law)
- Theorem 7.10.3 (uniform integrability and L^1 convergence)
- Theorem 9.5.16 (Ergodic Theorem; case of ergodic \mathbb{Q} only)
- Lemma 10.1.8 (renewal equation for $m(t)$)
- Theorems 10.2.1 and 10.2.3 (renewal theorems)
- Lemma 10.2.9 (Wald’s equation)
- Theorem 10.5.1 (renewal-reward theorem)
- Theorem 12.2.3 (Hoeffding’s inequality)
- Theorem 12.3.1 (the submartingale convergence theorem, only the case $\mathbb{E}|Y_0| < \infty$)
- Theorem 12.3.3 (Snell’s upcrossing inequality)
- Theorem 12.4.5 (stopping preserves submartingale property)
- Theorem 12.4.11 (optional sampling theorem)
- Theorem 12.5.1 (optional stopping theorem)
- Theorem 12.5.2 (optional stopping theorem for UI martingales)
- Theorem 12.5.9 (another optional stopping theorem)
- Theorem 12.6.1 (maximal inequality, submartingale case only)
- Theorem 12.7.4 (backward martingale convergence theorem)

Theorems from Mörters & Peres:

- Theorem 1.22 (nowhere monotonicity of BM)
- Proposition 1.23 (BM compared to \sqrt{n})
- Theorem 1.30 (nowhere Lipschitz continuity of BM)

Other Theorems:

- de Finetti’s Theorem (given the estimate on binomial coefficients in Exercise 3 on the sheet)