# TMA970 Inledande matematisk analys F1/TM1

## Lösningar 25/8 - 23

**1.** (a) konvergent; (b) konvergent;
(c) divergent; (d) falskt;
(e) sant; (f) sant.

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**2.** (a) $$0 < \left| \frac{\sqrt[3]{n^2}\,\sin(n!)}{n+1} \right| \leq \frac{n^{\frac{2}{3}}}{n+1} \xrightarrow[n\to\infty]{} 0$$

$$\Rightarrow \lim_{n\to\infty} \frac{\sqrt[3]{n^2}\,\sin(n!)}{n+1} = 0$$

(b) $$\frac{\sqrt{1+2x}-3}{\sqrt{x}-2} = \frac{(\sqrt{1+2x}-3)(\sqrt{x}+2)}{x-2^2} =$$

$$= \frac{(\sqrt{1+2x}-3)(\sqrt{1+2x}+3)(\sqrt{x}+2)}{(x-4)(\sqrt{1+2x}+3)} =$$

$$= \frac{(\underbrace{1+2x-9}_{2(x-4)})(\sqrt{x}+2)}{(x-4)(\sqrt{1+2x}+3)} = \frac{2(\sqrt{x}+2)}{\sqrt{1+2x}+3} \xrightarrow[x\to 4]{} \frac{4}{3}$$

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**3.** $$f(x) = \frac{1+x}{\sqrt{1-|x|}} \qquad D_f:\ 1-|x|>0 \Leftrightarrow -1<x<1$$

$f(x)=0$ för inga $x \in D_f$

$f(x)>0$ i $D_f$

ej jämn/udda, ej periodisk

$f(0)=1$

$\triangle 2$

$x \geq 0: \quad f(x) = \dfrac{1+x}{\sqrt{1-x}} \xrightarrow[x \to 1_-]{} +\infty$

$x < 0: \quad f(x) = \dfrac{1+x}{\sqrt{1+x}} = \sqrt{1+x} \xrightarrow[x \to -1_+]{} 0$

Linjen $x = 1$ är vertikal asymptot.

$x > 0: \quad f'(x) = \dfrac{\sqrt{1-x} - \frac{-1}{2\sqrt{1-x}}(1+x)}{1-x} =$

$= \dfrac{2(1-x) + (1+x)}{2(1-x)^{3/2}} = \dfrac{3-x}{2(1-x)^{3/2}} > 0$ i $(0,1)$

Högerderivata i 0: $\underline{\frac{3}{2}}$

$x < 0: \quad f'(x) = \dfrac{1}{2\sqrt{1+x}} > 0$ i $(-1,0)$

Vänsterderivata i 0: $\underline{\frac{1}{2}}$ $\quad \nexists f'(0)$

$\lim_{x \to -1_+} f'(x) = +\infty \Rightarrow$ linjen $x = -1$ vertikal tangent

(om man utökar med punkten $(-1, 0)$)

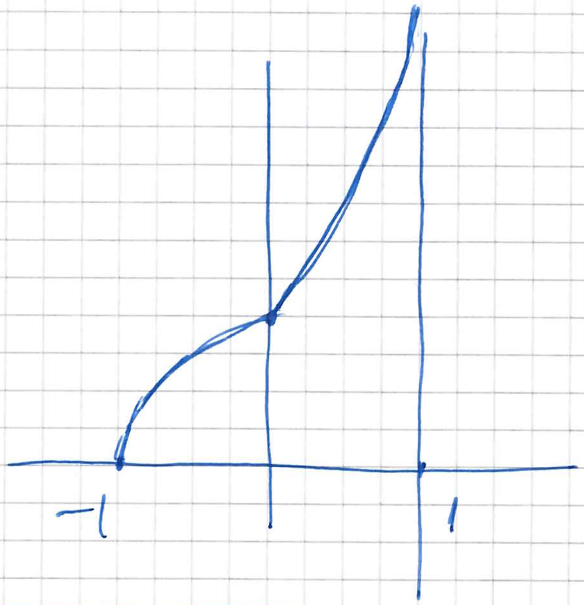
$x > 0: \quad f''(x) = \dfrac{-2(1-x)^{3/2} + 2 \cdot \frac{3}{2}(1-x)^{1/2}(3-x)}{4(1-x)^3} =$

$= \dfrac{\sqrt{1-x}\,(-2+2x+9-3x)}{4(1-x)^3} = \dfrac{7-x}{4(1-x)^{5/2}} > 0$ i $(0,1)$

$x < 0: \quad f''(x) = -\dfrac{1}{4} \cdot \dfrac{1}{(1+x)^{3/2}} < 0$ i $(-1,0)$

$\Rightarrow f$ konvex i $(0,1)$, konkav i $(-1,0)$

$\Rightarrow$ inflexion i $(0,1)$




**4.** (a) $$\int \frac{dx}{\cos^4 x} = \int \frac{1}{\cos^2 x} \cdot \frac{\sin^2 x + \cos^2 x}{\cos^2 x}\, dx =$$

$$= \int (1+\tan^2 x)\underbrace{(\tan x)'\, dx}_{= dt} = [t = \tan x]$$

$$= \int (1+t^2)\,dt = t + \frac{1}{3}t^3 (+C) =$$

$$= \tan x + \frac{1}{3}(\tan x)^3 (+C)$$

(b) $$\int_0^2 x e^{\sqrt{x}}\, dx = \left[\begin{array}{l} t = \sqrt{x} \\ x = t^2;\ dx = 2t\,dt \end{array}\right]$$

$$= \int_0^{\sqrt{2}} t^2 e^t \cdot 2t\, dt = 2\int_0^{\sqrt{2}} t^3 e^t\, dt =$$

$$= 2\left( \left[t^3 e^t\right]_0^{\sqrt{2}} - \int_0^{\sqrt{2}} 3t^2 e^t\, dt \right) =$$

$$= 2\left( 2\sqrt{2}\, e^{\sqrt{2}} - 3\left[t^2 e^t\right]_0^{\sqrt{2}} + 3\int_0^{\sqrt{2}} 2t \cdot e^t\, dt \right) =$$

$$= 2\left( 2\sqrt{2}\, e^{\sqrt{2}} - 6e^{\sqrt{2}} + 6\left[t e^t\right]_0^{\sqrt{2}} - 6\int_0^{\sqrt{2}} e^t\, dt \right) =$$

$$= 2e^{\sqrt{2}}\left(2\sqrt{2} - 6 + 6\sqrt{2}\right) - 2\cdot 6\, e^{\sqrt{2}} + 2\cdot 6 =$$

$$= \underbrace{2\cdot 6}_{= 12} + 4e^{\sqrt{2}}\left(4\sqrt{2} - 6\right) = 12 + 4e^{\sqrt{2}}\left(4\sqrt{2} - 6\right)$$

5. $f(x) = \arctan\left(\frac{1}{x^2} + \frac{1}{x-1}\right)$ 4

$D_f$ : $x \neq 0$ , $x \neq 1$

(arctan definierad i hela $\mathbb{R}$)

$\lim_{x \to 0_\pm} f(x) = \text{"}\arctan(+\infty)\text{"} = \frac{\pi}{2}$

$\lim_{x \to 1_-} f(x) = \text{"}\arctan(-\infty)\text{"} = -\frac{\pi}{2}$

$\lim_{x \to 1_+} f(x) = \text{"}\arctan(+\infty)\text{"} = \frac{\pi}{2}$

Olika vänster- och högergränsvärden i 1 $\Rightarrow$ f kan ej utvidgas till funktion, kontinuerlig i hela $\mathbb{R}$

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6. $x > a$ : $f'(x) = \varphi(x) + (x-a)\varphi'(x)$

$x < a$ : $f'(x) = -\varphi(x) + (a-x)\varphi'(x)$

Högerderivata i a : $\varphi(a)$

Vänsterderivata i a : $-\varphi(a)$

$\Rightarrow$ f deriverbar i a omm $\varphi(a) = 0$.