1

$\nabla f = \langle -2(y^7 - 3y)\sin(2x), (7y^6 - 3)\cos(2x) \rangle$

$\nabla f(\frac{\pi}{4}, 1) = \langle -2(1^7 - 3 \cdot 1)\sin\frac{\pi}{2}, (7-3)\cos\frac{\pi}{2} \rangle$

$= \langle 4, 0 \rangle$

$\bar{v} = \langle 2, 5 \rangle$

$|\bar{v}| = \sqrt{2^2 + 5^2} = \sqrt{29}$

$D_{\bar{u}} f(\frac{\pi}{4}, 1) = \nabla f(\frac{\pi}{4}, 1) \cdot \langle \frac{2}{\sqrt{29}}, \frac{5}{\sqrt{29}} \rangle$

$= \langle 4, 0 \rangle \cdot \langle \frac{2}{\sqrt{29}}, \frac{5}{\sqrt{29}} \rangle = \frac{8}{\sqrt{29}}$

a)

$\nabla f = \langle -2x, -6y+4 \rangle = 0.$

$x = 0, \quad y = \frac{2}{3}$

$(0, \frac{2}{3})$ is a critical pt.

$f_{xx} = -2, \quad f_{yy} = -6, \quad f_{xy} = 0.$

$D(0, \frac{2}{3}) = (-2)(-6) = 12 > 0.$

$f_{xx}(0, \frac{2}{3}) = -2 < 0.$

$\Rightarrow (0, \frac{2}{3})$ is a loc. max.

b). Abs. max/min are either on the boundary of D or at the critical pt $(0, \frac{2}{3}) \in D$.

$f(0, \frac{2}{3}) = -3 \cdot \frac{4}{9} + \frac{8}{3} - 1 = \boxed{\frac{1}{3}}.$

We use Lagrange Method to find abs. max/min on the boundary.

$$\begin{cases} \nabla f = \lambda \nabla g \\ g(x,y) = 0 \end{cases}$$

$g(x,y) = x^2 + 4y^2 - 25$

/2.

$$\begin{cases} \langle -2x, -6y+4 \rangle = \lambda \langle 2x, 8y \rangle \\ x^2+4y^2 = 25 \end{cases}$$

$$\begin{cases} \langle -x, -3y+2 \rangle = \lambda \langle x, 4y \rangle \\ x^2+4y^2 = 25 \end{cases}$$

$$\begin{cases} -x = \lambda x \quad (1) \Longrightarrow x = 0 \quad \text{or} \quad \lambda = -1. \\ -3y+2 = 4\lambda y \quad (2) \\ x^2+4y^2 = 25 \quad (3) \end{cases}$$

<u>case $x=0$</u> : From (3) find $4y^2 = 25$

$$y^2 = \frac{25}{4}$$

$$y = \pm \frac{5}{2}.$$

(From (2) can find λ, but we don't need it).

$$\left(0, \frac{5}{2}\right), \left(0, -\frac{5}{2}\right).$$

$$f\left(0, \frac{5}{2}\right) = -3 \cdot \frac{25}{4} + 10 - 1 = 9 - \frac{75}{4} = \boxed{-\frac{39}{4}}.$$

$$f\left(0, -\frac{5}{2}\right) = -3 \cdot \frac{25}{4} - 10 - 1 = -11 - \frac{75}{4} = \boxed{-\frac{119}{4}}.$$

<u>case $\lambda = -1$</u> : From (2) find $-3y+2 = -4y$

$$y = -2.$$

From (3): $x^2 + 16 = 25$

$x^2 = 9$, $x = \pm 3$.

$(3,-2)$ and $(-3,-2)$.

$$f(3,-2) = f(-3,-2) = -9-12-8-1 = \boxed{-30}.$$

Abs. min is -30.

Abs. max is $\frac{1}{3}$.

(3)

$$\iint_D y(x^2+y^2)\,dA = \int_0^{\pi}\int_1^{3} r\sin\theta \cdot r^2 \cdot r\,dr\,d\theta$$

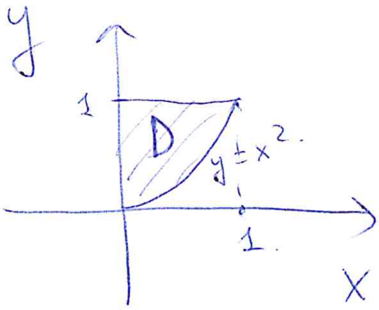
$$= \int_0^{\pi}\int_1^{3} r^4 \sin\theta \,dr\,d\theta = \int_0^{\pi} \sin\theta \left[\frac{r^5}{5}\right]_{r=1}^{r=3} d\theta$$

$$= \int_0^{\pi} \left(\frac{3^5}{5} - \frac{1}{5}\right) \sin\theta \,d\theta = -\frac{3^5-1}{5}\left[\cos\theta\right]_{\theta=0}^{\theta=\pi} =$$

$$= \frac{2(3^5-1)}{5}.$$

/5

4



$$\iiint_E \overset{x}{\cancel{(x-z)}}\, dV = \iint_D \int_0^{1+2x+y} \overset{x}{\cancel{(x-z)}}\, dz\, dA$$

$$= \iint_D \big[x\, z\big]_{z=0}^{z=1+2x+y}\, dA = \iint_D x(1+2x+y)\, dA$$

$$= \int_0^1 \int_{x^2}^1 \left(x + 2x^2 + xy\right) dy\, dx = \int_0^1 \left[xy + 2x^2y + \frac{xy^2}{2}\right]_{y=x^2}^{y=1} dx$$

$$= \int_0^1 \left(x + 2x^2 + \frac{x}{2} - x^3 - 2x^4 - \frac{x^5}{2}\right) dx$$

$$= \int_0^1 \left(\frac{3x}{2} + 2x^2 - x^3 - 2x^4 - \frac{x^5}{2}\right) dx$$

$$= \left[\frac{3x^2}{4} + \frac{2x^3}{3} - \frac{x^4}{4} - \frac{2x^5}{5} - \frac{x^6}{12}\right]_{x=0}^{x=1}$$

$$= \frac{3}{4} + \frac{2}{3} - \frac{1}{4} - \frac{2}{5} - \frac{1}{12} = \frac{41}{6}.$$

/6.

5

i)

$$\bar{r}(t) = \langle 5\cos(2t),\ 3t+1,\ 5\sin(2t) \rangle, \quad 0 \le t \le \pi$$

$$\bar{r}'(t) = \langle -10\sin(2t),\ 3,\ 10\cos(2t) \rangle$$

$$|\bar{r}'(t)| = \sqrt{100\sin^2(2t) + 9 + 100\cos^2(2t)} = \sqrt{109}.$$

$$L = \int_0^{\pi} |\bar{r}'(t)|\, dt = \pi\sqrt{109}.$$

ii) $$\text{work} = \int_C \bar{F} \cdot d\bar{r} = \int_C (-z)\,dx + e^y\,dy + x\,dz$$

$$= \int_0^{\pi} \Big(-5\sin(2t)\,(-10)\sin(2t) + e^{3t+1} \cdot 3 + 5\cos(2t) \cdot 10\cos(2t) \Big)\, dt$$

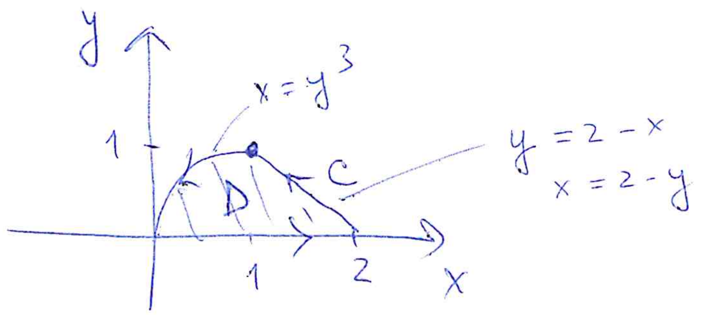
$$= \int_0^{\pi} \left(50\sin^2(2t) + 3e^{3t+1} + 50\cos^2(2t) \right) dt$$

$$= \int_0^{\pi} \left(50 + 3e^{3t+1} \right) dt = \Big[50t + e^{3t+1} \Big]_{t=0}^{t=\pi}$$

$$= 50\pi + e^{3\pi+1} - e.$$

7

(6).



$$\int_C \underbrace{x e^x}_{P}\, dx + \underbrace{\left(x^2 y - \sqrt{y - \cos^2 y}\right)}_{Q} dy$$

<u>Green's Th.</u> $\displaystyle\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$

$$= \iint_D 2xy\, dA = \int_0^1 \int_{y^3}^{2-y} 2xy\, dx\, dy$$

$$= \int_0^1 \left[x^2 y\right]_{x=y^3}^{x=2-y} dy = \int_0^1 \left((2-y)^2 - y^6\right) y\, dy$$

$$= \int_0^1 \left(4 - 4y + y^2 - y^6\right) y\, dy = \int_0^1 \left(4y - 4y^2 + y^3 - y^7\right) dy$$

$$= \left[2y^2 - \frac{4y^3}{3} + \frac{y^4}{4} - \frac{y^8}{8}\right]_{y=0}^{y=1} = 2 - \frac{4}{3} + \frac{1}{4} - \frac{1}{8}.$$

$$= \frac{48 - 32 + 6 - 3}{24} = \frac{19}{24}$$

8)

7

$$\overline{F} = \langle -z^2,\ x^2 e^{2z},\ z \rangle$$

$$\operatorname{div} \overline{F} = 0 + 0 + 1 = 1.$$

$$\iint_S \overline{F} \cdot d\overline{S} \ \underset{=\!=\!=}{\overset{\text{Gauss Th.}}{=}} \ \iiint_E 1 \cdot dV$$

$$= \int_0^{9/2} \int_0^{9-2x} \int_0^{3-\frac{2x}{3}-\frac{y}{3}} dz\, dy\, dx$$

$$= \int_0^{9/2} \int_0^{9-2x} \left(3 - \frac{2x}{3} - \frac{y}{3}\right) dy\, dx$$

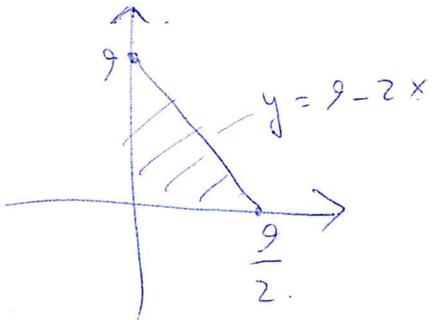
$$= \int_0^{9/2} \left[3y - \frac{2xy}{3} - \frac{y^2}{6} \right]_{y=0}^{y=9-2x} dx$$

$$= \int_0^{9/2} (9-2x)\left(3 - \frac{2x}{3} - \frac{9-2x}{6}\right) dx$$

$$= \int_0^{9/2} (9-2x)\left(\frac{3}{2} - \frac{x}{3}\right) dx$$

$$= \int_0^{9/2} \left(\frac{27}{2} - 6x + \frac{2x^2}{3} \right) dx = \left[\frac{27}{2}x - 3x^2 + \frac{2x^3}{9} \right]_{x=0}^{x=9/2}$$

$$= \frac{27}{2} \cdot \frac{9}{2} - \frac{3 \cdot 81}{4} + \frac{2 \cdot 9^3}{9 \cdot 8} = \frac{81}{4}(3 - 3 + 1) = \frac{81}{4}.$$



$$2x + y = 9$$
$$y = 9 - 2x$$

$$2x + y + 3z = 9$$
$$3z = 9 - 2x - y$$
$$\boxed{z = 3 - \frac{2x}{3} - \frac{y}{3}}$$