MVE 091 $\boxed{R3}$ (1)

<u>4.4:</u> $f(x) = \frac{1}{\log 2} \cdot \frac{1}{x} \quad 25 \le x \le 50.$

<u>L:</u> a) $f(x)$ är en tfhn om (sid 89)

1) $f(x) \ge 0$

2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

1) håller uppenbarligen

2) $\int_{-\infty}^{\infty} f(x)\,dx = \int_{25}^{50} \frac{1}{\log 2} \cdot \frac{1}{x}\,dx = \frac{1}{\log 2}\left[\log x\right]_{25}^{50}$

$= \frac{1}{\log 2}(\log 50 - \log 25) = \frac{1}{\log 2} \log \frac{50}{25} = 1$ //

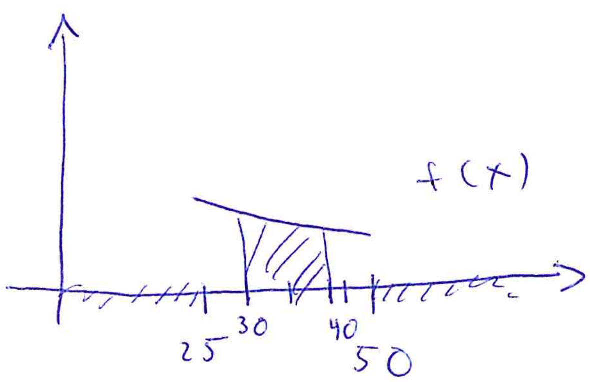
b) Vi har att $\mathbb{P}(a \le X \le b) = \int_a^b f(x)\,dx$

Här får vi att

$\mathbb{P}(30 \le X \le 40) = \int_{30}^{40} \frac{1}{\log 2} \cdot \frac{1}{x}\,dx = \frac{1}{\log 2}\left[\log x\right]_{30}^{40}$

$= \frac{1}{\log 2} \cdot (\log 40 - \log 30) = \frac{1}{\log 2} \cdot \log \frac{4}{3}$ //

c)



//

4.16: Låt

$$f(x) = \frac{1}{\log 2} \cdot \frac{1}{x} \qquad 25 \leq x \leq 50$$

Hitta väntevärde, varians & standardavvikelse.

L: Vi har att

$$E[X] = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{25}^{50} x \cdot \frac{1}{\log 2} \cdot \frac{1}{x}\,dx = \int_{25}^{50} \frac{1}{\log 2}\,dx$$

$$= 25 \cdot \log 2 \,//$$

$$\mathrm{Var}(X) = E[X^2] - E[X]^2 \quad \text{och}$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_{25}^{50} x^2 \frac{1}{\log 2} \cdot \frac{1}{x}\,dx = \frac{1}{\log 2} \int_{25}^{50} x\,dx$$

$$= \frac{1}{\log 2} \left[\frac{x^2}{2} \right]_{25}^{50} = \frac{1}{2 \log 2} (50^2 - 25^2) = \frac{2500 - 625}{2 \log 2}$$

$$= \frac{1875}{2 \log 2} \,//$$

Vi ser att $\mathrm{Var}(X) = \dfrac{1875}{2 \log 2} - (25 \log 2)^2$

$$\sigma = \sqrt{\mathrm{Var}(X)} = \sqrt{\frac{1875}{2 \log 2} - (25 \log 2)^2}$$

4,28: Visa att för $\alpha, \beta > 0$

$$\int_0^\infty \frac{1}{\Gamma(\alpha)\beta^\alpha} \cdot x^{\alpha-1} e^{-x/\beta} dx = 1$$

s.a. gamma-förd. verkligen är en förd.

L: $\Gamma(\alpha) = \int_0^\infty z^{\alpha-1} e^{-z} dz$

Insättning i $\int$ ovan kommer inte ge ngt. Om påståendet är sant så måste:

$$\Gamma(\alpha) = \int_0^\infty \frac{1}{\beta^\alpha} x^{\alpha-1} e^{-x/\beta} dx$$

VL beror ~~ej~~ på β

HL " på β !?

Substitution: $z = x/\beta$! $\Rightarrow dx = \beta dz$

$$\int_0^\infty \frac{1}{\beta^\alpha} x^{\alpha-1} e^{-x/\beta} dx = \int_0^\infty \frac{1}{\beta^\alpha} (z\beta)^{\alpha-1} e^{-z} \beta dz$$

$$= \int_0^\infty z^{\alpha-1} e^{-z} dz = \Gamma(\alpha) //$$

4.43: Vi har att $X \sim N(10, 9)$

L: a) Vi söker $P(X \leq 15)$. Vi transformerar och använder tabell. Kom ihåg:

Om $X \sim N(\mu, \sigma^2) \Rightarrow \frac{X-\mu}{\sigma} \sim N(0,1)$.

Här är $\mu = 10$, $\sigma = \sqrt{Var(X)} = \sqrt{9} = 3$

$$P(X \leq 15) = P\left(\frac{X-10}{3} \leq \frac{15-10}{3}\right) = \{Z \sim N(0,1)\}$$

$$= P(Z \leq 5/3) \approx P(Z \leq 1.666\ldots) \approx 0{,}952$$

↑ tabell sid 698 //

b) Vi söker t s.a.

$$0.05 = P(X \leq t) = P\left(\frac{X-10}{3} \leq \frac{t-10}{3}\right)$$

$$= P\left(Z \leq \frac{t-10}{3}\right)$$

Tabell (sid 69~~8~~) att $\frac{t-10}{3} \approx -1.645$

$\Rightarrow$ $t = 10 - 3 \cdot 1{,}645 \approx 5.065$ timmar //

4.60: $X \sim Wei(\alpha, \beta)$ med $\alpha = 0.04$, $\beta = 2$

a) Bestäm tfkn, $E[X]$ e $Var(X)$

L: Från föreläsningen:

$$f_X(x) = \alpha\beta x^{\beta-1} e^{-\alpha x^\beta} = 0.08 x^1 e^{-0.04x^2} \qquad x \geq 0$$

$$E[X] = \alpha^{-1/\beta}\Gamma(1 + 1/\beta) = 0.04^{-1/2}\Gamma(1 + 1/2)$$

$$= \frac{\Gamma(3/2)}{0.2}$$

$$Var(X) = \{\text{s. 1245}\} = \alpha^{-2/\beta}\Gamma(1 + 2/\beta) - E[X]^2$$

$$= 0.04^{-1}\Gamma(1+1) - \frac{\Gamma(3/2)^2}{0.04}$$

$$= \frac{1}{0.04}\left(\Gamma(2) - \Gamma(3/2)^2\right) //$$

b) ~~Hitta~~ Hitta överlevnadsfknen:

$$R_X(t) = 1 - F_X(t) = 1 - \int_{-\infty}^{t} f_X(s)\,ds$$

$$= 1 - \int_0^t \alpha\beta x^{\beta-1} e^{-\alpha x^\beta}\,dx = 1 - \left[-e^{-\alpha x^\beta}\right]_0^t$$

$$= 1 + e^{-\alpha t^\beta} - 1 = e^{-\alpha t^\beta} = e^{-0.04t^2} //$$

d) Hitta felintensiteten

$$\lambda_X(t) = \frac{f_X(t)}{R_X(t)} = \frac{\alpha\beta t^{\beta-1} e^{-\alpha t^\beta}}{e^{-\alpha t^\beta}} = \alpha\beta t^{\beta-1}$$

$$= 0.04 \cdot 2 \cdot t^{2-1} = 0.08 \cdot t //$$

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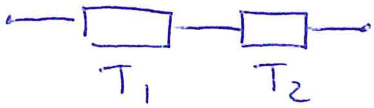
4.66: seriekoppling: (ober. komp.)



$T_1 \sim \text{Wei}(\underset{\alpha_1}{0.006}, \underset{\beta_1}{\cancel{\beta\neq}\,0.5}) \qquad T_2 \sim \text{Exp}(\underset{\beta_2}{0.00004})$

a) Bestäm överlevnadsfkner (vid 2500 h.)

L: Seriekopplat: $T = \min(T_1, T_2)$

Vi söker $R_T(t) = \mathbb{P}(T > t) = \mathbb{P}(\min(T_1, T_2) > t)$

$= \mathbb{P}(T_1 > t, T_2 > t) = \mathbb{P}(T_1 > t)\,\mathbb{P}(T_2 > t)$. (alt förel.)

$$\mathbb{P}(T_2 > t) = \int_t^\infty f_{T_2}(s)\,ds = \int_t^\infty \frac{1}{\beta_2} e^{-s/\beta_2} ds = \left[-e^{-s/\beta_2}\right]_t^\infty$$

$$= -0 + e^{-t/\beta_2} = e^{-t/\beta_2}.$$

$$\mathbb{P}(T_1 > t) = \int_t^\infty f_{T_1}(s)\,ds = \int_t^\infty \alpha_1 \beta_1 x^{\beta_1 - 1} e^{-\alpha_1 x^{\beta_1}} dx$$

$$= \left[-e^{-\alpha_1 x^{\beta_1}}\right]_t^\infty = -0 + e^{-\alpha_1 t^{\beta_1}} = e^{-\alpha_1 t^{\beta_1}}$$

$$\Rightarrow \mathbb{P}(T > t) = e^{-t/\beta_2} \cdot e^{-\alpha_1 t^{\beta_1}} = e^{-(t/0.00004 + 0.006\, t^{1/2})}$$ //

c) Som a) fast antag nu parallellkoppl.

L: parallell: $T = \max(T_1, T_2)$

$R_T(t) = 1 - F_T(t) = 1 - \mathbb{P}(T \leq t) = 1 - \mathbb{P}(\max(T_1, T_2) \leq t)$

$= 1 - \mathbb{P}(T_1 \leq t, T_2 \leq t) \underset{\substack{\uparrow \\ \text{ober.}}}{=} 1 - \mathbb{P}(T_1 \leq t)\,\mathbb{P}(T_2 \leq t)$

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$$\mathbb{P}(T_1 \le t) = \int_{-\infty}^{t} f_{T_1}(s)\,ds = \int_{0}^{t} \alpha_1 \beta_1 s^{\beta_1 - 1} e^{-\alpha_1 s^{\beta_1}}\,ds$$

$$= \left[-e^{-\alpha_1 s^{\beta_1}}\right]_0^t = 1 - e^{-\alpha_1 t^{\beta_1}}$$

$$\mathbb{P}(T_2 \le t) = 1 - \mathbb{P}(T_2 > t) = 1 - e^{-t/\beta_2}$$

$$\Rightarrow \quad R_T(t) = 1 - (1 - e^{-\alpha_1 t^{\beta_1}})(1 - e^{-t/\beta_2})$$

//