Ö6.1

den 5 oktober 2021 10:25

Uppgifter: A.10.2 → 30 / 31 sidan 584
A.10.3 → 4 / 6 sidan 592

30 Show that:
If $\vec{u}, \vec{v}, \vec{w}$ are any three mutually perpendicular unit vectors in $\mathbb{R}^3$ and $r = a\vec{u} + b\vec{v} + c\vec{w}$,
Then $a = \vec{r}.\vec{u}$, $b = \vec{r}.\vec{v}$ and $c = \vec{r}.\vec{w}$

Lösning: Enligt hypotes
$u \cdot u = 1 \quad v.v = 1 \quad w \cdot w = 1$
$u \cdot v = 0 \quad v \cdot w = 0$
$u \cdot w = 0$

$$\vec{r} = a\vec{u} + b\vec{v} + c\vec{w}$$

$$\vec{r} \cdot \vec{u} = (a\vec{u} + b\vec{v} + c\vec{w}) \cdot \vec{u} =$$ Enligt egenskapen: • är en assossiativa och distributiva operation.

$$= a\,\vec{u}\cdot\vec{u} + b\,\vec{v}.\vec{u} + c\,\vec{w}.\vec{u}$$
$$= a.1 + b.0 + c.0$$
$$\begin{cases} \vec{v}.\vec{u} = \vec{u}.\vec{v} = 0 \\ \vec{w}\cdot\vec{u} = \vec{u}.\vec{w} = 0 \end{cases}$$

Då $\boxed{\vec{r} \cdot \vec{u} = a}$

Med samma argument, beräknar vi

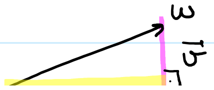
$$\vec{r}\cdot\vec{v} = (a\vec{u} + b\vec{v} + c\vec{w}) \cdot \vec{v} =$$
$$a\,\vec{u}\cdot\vec{v} + b\,\vec{v}\cdot\vec{v} + c\,\vec{w}\cdot\vec{v} =$$
$$a.0 + b.1 + c.0 = b \Rightarrow \boxed{\vec{r}.\vec{v} = b}$$

$$\vec{r}.\vec{w} = (a\vec{u} + b\vec{v} + c\vec{w}) \cdot \vec{w} =$$
$$a\vec{u}\cdot\vec{w} + b\vec{v}\cdot\vec{w} + c\vec{w}\cdot\vec{w} =$$
$$a.0 + b.0 + c.1 = c \qquad \boxed{\vec{r}\cdot\vec{w} = c}$$

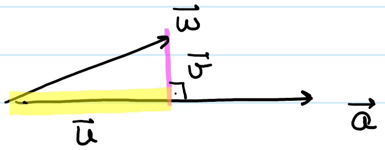
□

31 Resolving a vector in perpendicular directions
If $\vec{a}$ is a nonzero vector and $\vec{w}$ is any vector,
Find vectors $\vec{u}$ and $\vec{v}$ such that $w = \vec{u} + \vec{v}$
$\vec{u} \parallel$ to $\vec{a}$ and $\vec{v} \perp$ to $\vec{a}$.

Lösning $\qquad \vec{a} \neq \vec{0}$.



$\vec{w} = \vec{u} + \vec{v} \qquad \vec{u} \parallel \vec{a} \Rightarrow \vec{u} = \lambda\vec{a}$



$\vec{w} = \vec{u} + \vec{v}$ $\qquad$ $\vec{u} \parallel \vec{a} \Rightarrow \vec{u} = \lambda \vec{a}$

$\vec{v} \perp \vec{a} \Rightarrow \vec{v} \cdot \vec{a} = 0$

Enligt uppgift 30 / (projektion utveckling gört i Föreläsning)

Då $\vec{w} = \vec{u} + \vec{v} = \lambda \vec{a} + \vec{v}$

$\vec{w} \cdot \vec{a} = \vec{u} \cdot \vec{a} + \vec{v} \cdot \vec{a}$ $\qquad (\vec{v} \perp \vec{a})$

$\vec{w} \cdot \vec{a} = \lambda \vec{a} \cdot \vec{a} + \vec{0}$

$\Rightarrow \lambda = \dfrac{\vec{w} \cdot \vec{a}}{\vec{a} \cdot \vec{a}}$ $\qquad$ Då är $\vec{w} = \mathrm{proj}_{\vec{a}} \vec{w} + \vec{v}$

$\vec{w} = \dfrac{(\vec{w} \cdot \vec{a})\vec{a}}{(\vec{a} \cdot \vec{a})} + \vec{v}$

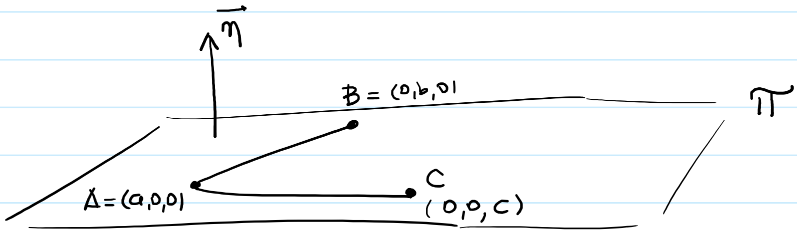
$$\boxed{\vec{v} = \vec{w} - \mathrm{proj}_{a} \vec{w}} \quad \square$$

A 10.3   Sidan 592

4) 1) Find a unit vector (versor) perpendicular to the plane containing the Points $A = (a,0,0)$, $B = (0,b,0)$ $C = (0,0,c)$

2) What is the area of the triangle with these 3 vertices?

4.1



$\vec{\eta} \perp \pi$ $\qquad$ $\vec{\eta}$ = normal vector.

$\vec{u} = \vec{AB} = (0,b,0) - (a,0,0) = (-a, b, 0)$

$\vec{v} = \vec{AC} = (0,0,c) - (a,0,0) = (-a, 0, c)$

} 2 riktnings vektors av planet π.

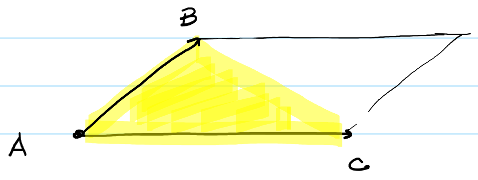
$$\vec{\eta} = \vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix} =$$

$$\vec{i}\,(b.c-0) - \vec{j}\,(-ac-0) + \vec{k}\,(ab) =$$

$$(bc)\,\vec{i} + ac\,\vec{j} + ab\,\vec{k} = \vec{n}$$

$$\hat{n} = \frac{\vec{n}}{\|n\|} = \frac{(bc,\ ac,\ ab)}{\sqrt{(bc)^2+(ac)^2+(ab)^2}}$$

2)



$$A_{\bar{u},\bar{v}}^{\text{parallelogram}} = \|\bar{u}\times\bar{v}\|$$

$$A_{\bar{u},\bar{v}}^{\text{triangle}} = \frac{1}{2}A^{\text{parallelogram}} = \frac{1}{2}\|\bar{u}\times\bar{v}\|$$

(6) Find a unit vector with positive $\vec{k}$ component that is perpendicular to both $2i-\vec{j}-2k$ och $2i-3j+k$

Lösning:

$\bar{u} = (2,-1,-2)$

$\vec{v} = (2,-3,1)$

$$\bar{w}\perp\bar{u} \wedge \bar{w}\perp\bar{v} \Leftrightarrow \bar{w} \,/\!/\, \bar{u}\times\bar{v}$$

$$\bar{u}\times\bar{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & -2 \\ 2 & -3 & 1 \end{vmatrix} = \vec{i}\,(-1-6) - \vec{j}\,(2+4) + \vec{k}\,(-6+2) =$$

$$\bar{u}\times\bar{v} = (-7,-6,-4)$$

$$-\bar{u}\times\bar{v} = (7,6,4)$$

$$\begin{cases} \vec{w} = \dfrac{-\bar{u}\times\bar{v}}{\|\bar{u}\times\bar{v}\|} = \dfrac{1}{\sqrt{49+36+16}}(7,6,4) = \dfrac{1}{\sqrt{101}}(7,6,4) \\ \|\vec{w}\| = 1. \end{cases}$$