Lecture 3: Bayes theorem and discrete distributions

MVE055 / MSG810
Mathematical statistics and discrete mathematics

Moritz Schauer

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GU & Chalmers University of Technology

Conditional distribution

If we know some event B occurs, the probability of A given the new information B can be calculated as follows:

Conditional probability

Assume that $\mathsf{P}(B) > 0$. The conditional probability of A given B is defined as

$$\mathsf{P}(A \mid B) = \frac{\mathsf{P}(A \cap B)}{\mathsf{P}(B)}. \tag{0.1}$$

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Multiplication rule for probabilities

For events A and B it holds

$$\mathsf{P}(A \cap B) = \boxed{}$$

The multiplication rule is useful to calculate probabilities of multiple events affecting each other.

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$$\mathsf{P}(A \cap B) = \mathsf{P}(B \mid A)\mathsf{P}(A) = \mathsf{P}(A \mid B)\mathsf{P}(B).$$

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$$\mathsf{P}(A \mid B) = \frac{\mathsf{P}(B \mid A)\mathsf{P}(A)}{\mathsf{P}(B)}$$

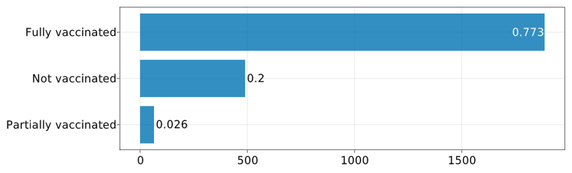
Often it is useful to rewrite the denominator $\mathsf{P}(B)$

$$\mathsf{P}(B) = \mathsf{P}(B \mid A)\mathsf{P}(A) + \mathsf{P}(B \mid A^c)\mathsf{P}(A^c)$$

Base rate fallacy

Base rate fallacy

In the fourth wave (July 10 - August 16, 2021) about 2400 (or 0.825 %) people 16 or older in Island have been diagnosed with Covid-19:



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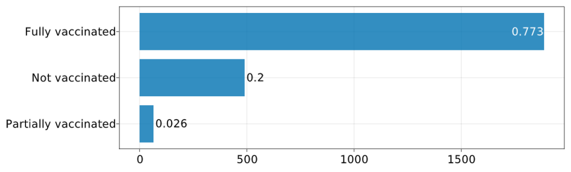
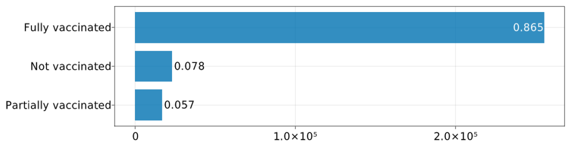


But (young) adults in Iceland's population are highly vaccinated



$$\mathrm{P}(\text{diagn} \mid \text{vacc}) = \frac{\mathrm{P}(\text{vacc} \mid \text{diagn})\mathrm{P}(\text{diagn})}{\mathrm{P}(\text{vacc})} = \frac{0.773 \cdot 0.00825}{0.864} = 0.00738$$

Base rate fallacy

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$$\mathrm{P}(\mathsf{diagn} \mid \mathsf{not\ vacc}) = \frac{0.200 \cdot 0.00825}{0.0783} = 0.0211$$

Base rate fallacy

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$$\mathrm{P}(\textsf{diagn} \mid \textsf{not vacc}) = \frac{0.200 \cdot 0.00825}{0.0783} = 0.0211$$

$$\mathrm{P}(\textsf{diagn} \mid \textsf{part. vacc}) = \frac{0.0262 \cdot 0.00825}{0.0570} = 0.00379 \text{ (}\textsf{sic!}\text{)}$$

Independent events

Two events A and B are independent if knowing whether B occured does not change the probability of A

$$\mathsf{P}(A \mid B) = \mathsf{P}(A).$$

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Simple example: Throw a 6-sided die. Are $A = \{5, 6\}$ and $B = \{1, 3, 5\}$ dependent?

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Simple example: Throw a 6-sided die. Are $A = \{5, 6\}$ and $B = \{1, 3, 5\}$ dependent?

$$\mathrm{P}(A)\mathrm{P}(B) = \frac{2}{6}\frac{3}{6} = \frac{1}{6}, \quad \mathrm{P}(A \cap B) = \mathrm{P}(\{5\}) = \frac{1}{6}.$$

If I tell you A happened, that does not change probabilities of B: $\mathrm{P}(B \mid A) = \mathrm{P}(B) = \frac{3}{6}$.

Random variables

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Example: X is the number of eyes on a 6-sided die.

We denote random variables with capital letters, often X or Y.

Example:

Pair of dice

Throw a pair of dice, count the total number of eyes, call that random variable X. Consider the **event** that $X = 7$.

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Event? What are the actual ω making our event and sample space? You could take

$$A = \{\epsdice{1}\epsdice{6}, \epsdice{2}\epsdice{5}, \epsdice{3}\epsdice{4}, \epsdice{4}\epsdice{3}, \epsdice{5}\epsdice{2}, \epsdice{6}\epsdice{1}\}, \quad \Omega = \{\epsdice{1}\epsdice{1}, \ldots, \epsdice{6}\epsdice{6}\}$$

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$$\mathrm{P}(X = 7) = \mathrm{P}(A) = \frac{|A|}{|\Omega|} = \frac{6}{36}$$

Pair of dice

Throw a pair of dice, count the total number of eyes, call that random variable X. Consider the **event** that $X = 7$.

Event? What are the actual ω making our event and sample space? You could take

$$A = \{⚀⚅, ⚁⚄, ⚂⚃, ⚃⚂, ⚄⚁, ⚅⚀\}, \quad \Omega = \{⚀⚀, \ldots, ⚅⚅\}$$

$$\mathrm{P}(X = 7) = \mathrm{P}(A) = \frac{|A|}{|\Omega|} = \frac{6}{36}$$

Value k	2	3	4	5	6	7	8	9	10	11	12
Probability $\mathrm{P}(X{=}k)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Pair of dice

The following holds for $k \in \{2, \dots, 12\}$:

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Check:

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Probability $\mathrm{P}(X{=}k)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	0

Discrete random variables

Discrete random variables

A random variable is called discrete if it is integer-valued or otherwise has only a finite or countable number of values.

Example: $Y = X/2$ is discrete (but can take non-integers such as $Y = 5.5$ as values.)

Probability mass function

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Sometimes we write f_X to talk about X's own probability mass function.

Sum of two dice

$$f(k) = \begin{cases} \frac{6-|k-7|}{36} & \text{if } k \in \{2, 3, \ldots, 12\} \\ 0 & \text{otherwise} \end{cases}$$

is the probability mass function for the random variable which counts the sum of two dice.

Two coins

Flip two coins... count the number of heads. Call it X.

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$f(0) = \frac{1}{4}, f(1) = \frac{1}{2}$ and $f(2) = \frac{1}{4}$

$f(x) = 0$ otherwise if $x \notin \{0, 1, 2\}$.

Flip two coins... count the number of heads. $f_X(0) = \frac{1}{4}$, $f_X(1) = \frac{1}{2}$ and $f_X(2) = \frac{1}{4}$.

What is $\mathrm{P}(X \in \{1, 2\}) = \mathrm{P}(1 \leqslant X \leqslant 2)$?

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Let $Y = X/2$. What is $\mathrm{P}(Y > 0)$?

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Rule

For integer valued X

$$\mathsf{P}(m \leqslant X \leqslant n) = \sum_{k=m}^{n} f(k)$$

for any integers m and n.

Probability mass function

Not all functions are probability mass functions. Because they describe probability distributions, some conditions must hold.

$f(k)$ is a probability mass function if and only if

- $f(k) \geqslant 0$ for all k.
- $\sum_{\text{all } k} f(k) = 1$.

If somebody gives you a probability mass function, there is a random variable for it.

Distribution function

Assume X is a discrete random variable. Its distribution function is given by

$$F(x) = \mathsf{P}(X \leqslant x) = \sum_{k \leqslant x} f_X(k),$$

Flip two coins... count the number of heads. Call it X.
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$$\begin{aligned}
F(0) &= f(0) = \tfrac{1}{4} \\
F(1) &= f(0) + f(1) = \tfrac{1}{4} + \tfrac{1}{2} \\
F(2) &= f(0) + f(1) + f(2) = 1
\end{aligned}$$

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What is the probability to throw k times heads in a row with a fair coin?

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$$f(0) = \frac{1}{2}, \quad f(1) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}, \quad f(2) = \frac{1}{8}, \quad f(k) = \left(\frac{1}{2}\right)^{k+1}$$

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$$P(X > 0) = f(1) + f(2) + f(3) + \ldots = \boxed{}$$

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Distribution function

For $F(x)$ it holds

- $F(x)$ is increasing
- $F(x) \to 1$ for $x \to \infty$.
- $F(x) \to 0$ for $x \to -\infty$.

Distribution function

Also

- $\mathsf{P}(a < X \leqslant b) = F(b) - F(a)$.
- $\mathsf{P}(X > a) = 1 - F(a)$.
- For integer valued random variables:
  $f(m) = F(m) - F(m-1)$.

We are often interested in the "average" outcome of a random variable.

Expected value

The expected value of a random variable is defined as

$$\mathsf{E}(X) = \sum_{\text{all } k} k f_X(k) \qquad \text{if } X \text{ is discrete,}$$

Recall: the average using fractions

Data set: grades of 24 students

$$5, 5, 6, 5, 6, 6, 6, 5, 5, 7, 6, 7, 5, 5, 5, 6, 6, 6, 5, 6, 5, 7, 6, 7$$

Table:

grade	$x_1 = 7$	$x_2 = 6$	$x_3 = 5$
fraction of students	$p_1 = 4/24$	$p_2 = 10/24$	$p_3 = 10/24$

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Average One can write the average in different forms

$$\text{Average} = \frac{5+5+6+\cdots+5+7+6+7}{24}$$

$$= \frac{7\cdot 4 + 6\cdot 10 + 5\cdot 10}{24} = 7\cdot\frac{4}{24} + 6\cdot\frac{10}{24} + 5\cdot\frac{10}{24} = \sum_{i=1}^{3} x_i \cdot p_i$$

Expected value

The expected value of a discrete random variable X with finitely many outcomes can also be written as

$$\mu = \mathrm{E}(X) = \sum_{\text{all } k} x_k \cdot \underbrace{\mathrm{P}(X = x_k)}_{f(x_k)}$$

$$= x_1 \cdot \mathrm{P}(X = x_1) + x_2\, \mathrm{P}(X = x_2) + \cdots + x_n \cdot \mathrm{P}(X = x_n)$$

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$$= x_1 \cdot \mathrm{P}(X = x_1) + x_2 \, \mathrm{P}(X = x_2) + \cdots + x_n \cdot \mathrm{P}(X = x_n)$$

Here x_i are the n possible outcomes and $P(X = x_i)$ are the probabilities of each outcome.

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$\mathsf{E}(X) = \boxed{0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} = 1}$

Rules for computing expected values

For the expected value,

- $\mathsf{E}(a) = a$.
- $\mathsf{E}(aX) = a\mathsf{E}(X)$.
- $\mathsf{E}(aX + b) = a\mathsf{E}(X) + b$.
- $\mathsf{E}(X + Y) = \mathsf{E}(X) + \mathsf{E}(Y)$.

Here X and Y are any two random variables and a and b are constants.

Transformations

If we transform the random variables by a function h we have:

Theorem ♡

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Coin example (with $h(x) = x/2$):

$$\mathsf{E}(X/2) = \frac{0}{2} \cdot f_X(0) + \frac{1}{2} \cdot f_X(1) + \frac{2}{2} \cdot f_X(2) = \tfrac{1}{2}$$

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$$= (\mathsf{E}(X))/2$$

Common distributions

Bernoulli distribution

The Bernoulli distribution describes a random experiment that can either succeed (with probability p) or fail (with probability $1 - p$.) Suppose we make a random experiment which succeeds with probability p and set

$$X = \begin{cases} 1, & \text{if the experiment succeeds} \\ 0, & \text{in case of failure.} \end{cases}$$

We have $f(1) = p$ and $f(0) = 1 - p$.

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We have $f(1) = p$ and $f(0) = 1 - p$.

Sometimes useful to write as $f(k) = p^k(1-p)^{1-k}$ for $k \in \{0, 1\}$.

The binomial distribution

Bernoulli distribution

A random variable X is Bernoulli distributed if it has probability mass function $f(1) = p$ and $f(0) = 1 - p$ and $= 0$ otherwise. We write $X \sim \mathrm{Ber}(p)$.

Examples?

The binomial distribution

The binomial distribution describes the probability of having exactly k successes in n independent Bernoulli trials with probability of success p.

If X is Binomial with parameters n and p we write:

$$X \sim \mathrm{Bin}(n, p)$$

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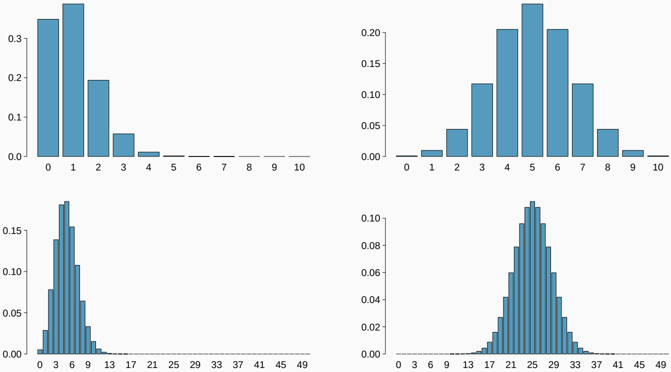
Ha, the sum of two coins with sides 0 and 1 is $\mathrm{Bin}(2, 0.5)$ distributed.

The binomial distribution



$n = 10$

$n = 50$

$p = 0.1$ $p = 0.5$

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Binomial distribution

A random variable X is Binomial distributed with parameters n, p if

$$\mathrm{P}(X = k) = \binom{n}{k} p^k (1-p)^{n-k} \qquad \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Sum of binomial distributed random variables

Sum of binomial distributed random variables.

If $X_1 \sim \mathrm{Bin}(n, p)$ and $X_2 \sim \mathrm{Bin}(m, p)$ are independent, then $X_1 + X_2 \sim \mathrm{Bin}(m + n, p)$.

("Dropping m items, couting the broken ones, dropping n more items, counting the additional broken ones is the same as dropping $m + n$ items...")

Geometric distribution

The experiment consists of a series of independent Bernoulli trials with probability of success equal to p.

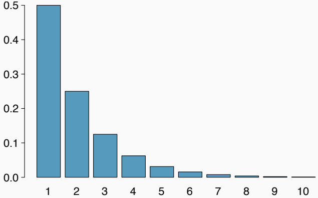
The random variable X denotes the number of trials needed to get the first success.

p is called the parameter of X.

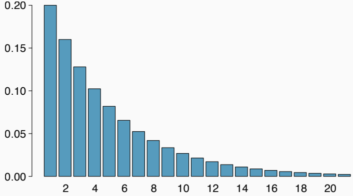
The geometric distribution

The geometric distribution describes the probability distribution of the number of trials needed k to get the first success, for a single event succeeding with probability p. ($k-1$ failures and 1 success.)

$p = 0.5$



$p = 0.2$



The geometric distribution

Geometric distribution

A random variable X is geometrically distributed with parameters p if

$$\mathrm{P}(X = k) = (1 - p)^{k-1} p, \quad k = 1, 2, \ldots$$

We write $X \sim \mathrm{Geom}(p)$.