Solution GS ch. 8.

8.1.10 $X = 0, 1, \ldots, n$ $\quad E[X] = V[X] = 1$

Chebyshev's inequality: $P(|X-\mu| \geq \varepsilon) \leq \frac{V(X)}{\varepsilon^2}$.

$$\Rightarrow P(|X-1| \geq \varepsilon) \leq \frac{1}{\varepsilon^2}.$$

Let $\varepsilon = k \geq 1$ (positive integer)

$$P(|X-1| \geq k) \leq \frac{1}{k^2}.$$

$$P(|X-1| \geq k) = P(X-1 \geq k) + P(X-1 \leq -k)$$
$$= P(X \geq k+1) + P(X \leq 1-k)$$

$$\Rightarrow P(X \geq k+1) \leq P(X \geq k+1) + P(X \leq 1-k) \leq \frac{1}{k^2}.$$

$$\left(\text{note that } P(X \leq 1-k) = \begin{cases} P(X=0) & \text{if } k=1 \\ 0 & \text{otherwise} \end{cases} \right)$$

8.2.4 X exponentially distributed with $\lambda = 0{,}1$.

$f(x) = \lambda e^{-\lambda t}$

(a) $E[X] = \frac{1}{\lambda} = 10$, $\quad V[X] = \frac{1}{\lambda^2} = 100$.

(b) $P(|X-10| \geq 2) \leq \frac{100}{4} = 25$ but probabilities are always ≤ 1

$$\Rightarrow P(|X-10| \geq 2) \leq 1$$

$P(|X-10| \geq 5) \leq \frac{100}{25} = 4 \Rightarrow P(|X-10| \geq 5) \leq 1$

$P(|X-10| \geq 9) \leq \frac{100}{81} \Rightarrow P(|X-10| \geq 5) \leq 1$

$P(|X-10| \geq 20) \leq \frac{100}{400} = 0{,}25$.

$$P(|X-10| \geq 2) = P(X-10 \geq 2) + P(X-10 \leq -2)$$
$$= P(X \geq 12) + P(X \leq 8)$$
$$= 1 - P(X < 12) + P(X \leq 8)$$
$$= 1 - F(12) + F(8) = 1 - (1 - e^{-1,2}) + 1 - e^{-0,8}$$
$$= 1 - e^{-0,8} + e^{-1,2} = 0,852$$

$$P(|X-10| \geq 5) = P(X-10 \geq 5) + P(X-10 \leq -5)$$
$$= P(X \geq 15) + P(X \leq 5)$$
$$= e^{-1,5} - e^{-0,5} + 1 = 0,617$$

$$P(|X-10| \geq 9) = P(X-10 \geq 9) + P(X-10 \leq -9)$$
$$= P(X \geq 19) + P(X \leq 1)$$
$$= e^{-1,9} - e^{-0,1} + 1 = 0,245$$

$$P(|X-10| \geq 20) = P(X-10 \geq 20) + P(X-10 \leq -20)$$
$$= P(X \geq 30) + \underbrace{P(X \leq -10)}_{=0 \text{ since } X \geq 0}$$
$$= 0,05.$$